Housing and Fertility*

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Abstract

I build a dynamic equilibrium model of household behavior with unobserved heterogeneity in the desired number of children to examine how policies targeting the housing market affect choices of fertility, location, and house size of young households. I estimate the model's structural parameters using data from Hungary to evaluate the dynamic effects of the Family Housing Allowance policy, which provided a sizeable lump-sum subsidy for house purchases, with built-in commitment regarding the number of children the family would have. The model suggests that the combination of lower interest rates and the allowance increases house prices substantially compared to the baseline, which for poorer households counteract some of the positive welfare effects of the policy. While according to the model, completed fertility increases due to the policy by around 5-10% on average, mainly driven by poorer households, their housing conditions worsen in the long run due to the elevated house prices. Richer households experience no adverse effects of the policy, however, their completed fertility remains unaffected.

Keywords: Housing Demand, Fertility, Housing Policy, Residential Choice

JEL Classification: J11, J13, J61, H31, R21

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1 Introduction

Housing plays a central role in the life of a typical household. In the United States and other developed countries, housing wealth has accounted for around 90% of young and poor households' assets in the last few decades and 35% of all household assets (Davis and Van Nieuwerburgh, 2015). Jordà et al. (2019) documents that housing has historically provided the largest part of capital stock, with returns comparable to equity. Furthermore, housing expenses have constituted around 20-25% of the total household expenses, a consistent finding for different periods and locations (Ortalo-Magné and Rady, 2006; Davis and Ortalo-Magné, 2011; Piazzesi and Schneider, 2016). Housing also contributed significantly to the Great Recession of 2008 (Mian et al., 2013; Famiglietti et al., 2020) and is understood to be a significant channel of monetary policy (Di Maggio et al., 2017). Real estate differs from other assets in several aspects. Houses are illiquid due to search frictions (Merlo et al., 2015); they are indivisible and relatively expensive, so most households must borrow to purchase one. Households usually end up owning only one house, where they reside and search for employment in its proximity (Davis and Van Nieuwerburgh, 2015). Compared to generic goods and services, housing is unique as its residents must consume its services. Furthermore, real estate functions as collateral to other types of borrowing, providing a channel through which household consumption is connected to the housing market (Attanasio et al., 2012; Di Maggio et al., 2017; Cloyne et al., 2019).

In the last few decades, the housing conditions of young households have been deteriorating. House ownership amongst young people has declined or been postponed notably in the developed world (Fisher and Gervais, 2011; Green and Lee, 2016; Flynn, 2020). Due to their high price, most households typically use mortgages to purchase houses. These mortgages are naturally mainly issued to young families who decide to suppress consumption in their early earning years to afford the down payment for the house (Chan et al., 2015). It is, however, necessary: most renters cannot afford a house. Therefore down payment, ranging between 20%-50% of the total price, is the chief obstacle for young households' home ownership (Davis and Van Nieuwerburgh, 2015). Down payment depends on house prices, which means that escalating house prices could contribute mainly to young households' inability to get a mortgage (Barakova et al., 2014). Moreover, evidence shows that housing is deeply intertwined with fertility decisions and the choice of residential location. Difficult housing conditions in terms of prices or security are documented to decrease or delay fertility amongst young households who look for their first home ownership, a wide-spread finding in the literature (Ermisch, 1999; Ström, 2010; Öst, 2012; Vignoli et al., 2013; Kulu and Steele, 2013; Dettling and Kearney, 2014; Day and Guest, 2016; Lin et al., 2016; Öst and Wilhelmsson, 2019)[1]. Young couples are also found to adjust their housing status to budgetary circumstances, credit constraints, and anticipated fertility (Ortalo-Magné and Rady, 1999; Mulder and Lauster, 2010; Fisher and Gervais, 2011; Ermisch and Steele, 2016; Vidal et al., 2017; Mulder, 2018). This channel, alongside essential changes in the female role of the household (Esping-Andersen and Billari, 2015), could have contributed to low fertility rates in developed countries (Billari and Kohler, 2004), as high home ownership along with low access to mortgages is associated with the lowest fertility rates (Mulder and Billari, 2010). An unfriendly housing environment for young couples could then contribute to potential future difficulties in the social security systems of European welfare states (Flynn, 2017; Zeman et al., 2018).

In this paper, I study how fertility and the housing market interplay with the residential choices of young households and how policy interventions that target housing conditions could effectively improve fertility outcomes. To address this question, I build a life-cycle model of forward-looking households choosing fertility, house size, ownership, and location, with unobserved heterogeneity in their desired number of children. I apply the model to analyze the effects of a government program in Hungary running since 2014/2015, called the Family Housing Allowance (further referred to as FHA)[2], which aimed to ease the borrowing constraints of young households at home purchases or mortgage down payments, and to revitalize the housing market of the country. The policy provides a substantial non-refundable lump sum at house purchases to families if they commit to or already have at least three children.[3] The overall effects are not straightforward to assess either in the long run or short

[1]At the same time, increasing house prices provide one of the most important source of asset value growth for households (Jordà et al., 2019).

[2]In Hungarian 'Családi Otthonteremtési Kedvezmény', abbreviated as CSOK

[3]The program since then has been extended to two children, with a substantially lower amount per child.

run (Banai et al., 2019; HNB, 2019). On the one hand, addressing the credit constraints of young households should lead to better housing conditions and fertility outcomes, especially considering direct incentives in this case. On the other, in an environment with an inelastic housing supply, the policy could result in a substantial increase in house prices which could force some families out of specific real estate markets, decreasing their welfare, and might result in the redistribution of resources between different types of households.

Due to the long-term nature of both mortgage and fertility decisions, dynamic models are the natural choice in the economic literature to analyze them, as fertility decisions and residential choices are jointly determined, possibly evolving with the housing markets in an endogenous manner. As mentioned, the housing market is documented to influence household welfare and decisions regarding fertility. At the same time, the other direction of causality is also present, as activity in the housing market is similarly affected by fertility decisions. Relatively higher mobility in the spatial dimension is well-documented for younger households, in which childbirth plays an instrumental role. Families look for areas with better amenities (Gambaro et al., 2017), which could indeed result in better life outcomes for small children (Chetty et al., 2016). Moving to agglomerations of larger cities can provide better amenities and labor market conditions, while it might increase the costs of housing, transportation, and consumption (Davis et al., 2014; Combes et al., 2019). It is also documented that families with different fertility choices can be observed to sort into different types of housing, more spacious dwellings giving a home to families with more children (Kulu and Vikat, 2007; Kulu and Steele, 2013; Chudnovskaya, 2019). Capturing endogenous relations of fertility and housing with dynamic structural models is critically important when we examine policies such as the Family Housing Allowance, as the reduced form approach cannot capture long-term unintended, potentially substantial consequences. An example of this issue, as studied by Parent and Wang (2007), is a benevolent expansion of the Canadian family tax exemption, which seemed to increase fertility in the short run. Still, the cohort-level long-run analysis shows that completed fertility remained unchanged, so the actual effect of the policy appeared only in the timing of births.

According to the model developed in this paper, the Family Housing Allowance (FHA) itself would not have increased house prices in the realized magnitude if the earlier somewhat higher interest rate conditions had stayed intact. However, I find that the combination of low interest rates and the housing allowance would result in an around 70-90% increase in house prices in the medium-run (4-6 years) both in the urban and rural areas. So while the FHA by itself would have resulted in higher ownership rates and fertility compared to the baseline, homeownership declines slightly amongst the poorer households due to elevated house prices. Consequently, according to the model specification, they are often forced to rent smaller, central-location apartments despite their adverse welfare effects. Richer families' housing conditions are not affected by the policy. Regarding completed fertility, I find that the policy changes the timing and completed fertility as well: births occur earlier in the female life-cycle, and around 5-10% increase in completed fertility is implied by the model, which seems to be high considering past results on the effect of welfare policies in Hungary (Spéder et al., 2020). The fertility effects of the policy are driven by families with lower education levels (of around 6-8%). However, the model also indicates an around 2.5% increase in births for families of higher education level. So overall, the policy seems to be effective in its goals regarding fertility, but it appears to damage the prospects of poorer households regarding housing welfare.

This paper primarily contributes to the literature by constructing a life-cycle model of households' joint decisions over fertility, housing size, and residential location with unobserved heterogeneity regarding the desired number of children and the endogenous evolution of house prices. It is the first attempt, as far as I know. However, in demography and economics, many stylized facts and descriptive evidence have been gathered over the years, suggesting strong interconnectedness between fertility, housing type, and location choice (Ermisch, 1999; Kulu and Vikat, 2007; Öst, 2012; Vignoli et al., 2013; Mulder, 2013; Dettling and Kearney, 2014; Day and Guest, 2016; Chudnovskaya, 2019). In this effort, I attempt to synthesize the relevant findings of macro, urban, and labor economics literature and demography on fertility, housing, and location choice. In the macroeconomic literature, authors mainly focused on studying houses as unique assets within the frames of optimal portfolio decisions, treating fertility as an issue of primarily exogenous consumption commitment (Cocco, 2005; Yao and Zhang, 2005; Love, 2010; Li et al., 2016; Fischer and Khorunzhina, 2019), endogenous fertility appears rarely and not in the context of housing (Sommer, 2016).

The effect of housing on the life-cycle of young households has been studied (Li and Yao, 2007; Attanasio et al., 2012), along with how migration and home ownership interact, impacting the welfare of households (Oswald, 2019). Still, these pieces did not consider the role of endogenous fertility in housing choices. In urban economics, work has been done on the joint modeling of housing services and location choice (Ortalo-Magné and Rady, 2006; Ortalo-Magné and Prat, 2016), along with how demographic changes interplay with urban costs (Combes et al., 2019). Similarly to the macroeconomic literature, fertility has been a less studied factor in urban economics. However, in labor economics, endogenous fertility plays a central role in the study of female labor supply (Becker, 1991), with substantial consequences for long-term labor market outcomes for women (Adda et al., 2017; Eckstein et al., 2019). I have not found examples of housing aspects as choice variables. Furthermore, the model enables the study of the effect of such policies in the medium and long run, which would not be possible in a reduced form setting so close in time to the start of the policy, while also connecting to the literature of combining structural models to study the effects of policy shocks, using the housing market as an ex-post model validation relating to the idea of Todd and Wolpin (2006).

The structure of the paper is as follows. First, I describe the policy context of the question in Hungary. Then I introduce the model in detail. Afterward, I turn to the empirical strategy of the paper, and I describe the estimation and calibration of the parameters. And finally, I use the model to run counterfactual policy experiments to evaluate the effects on fertility and housing.

2 Context: the Family Housing Allowance program of Hungary

Since 2010, the Hungarian government has introduced several new policies targeting directly or indirectly the fertility decisions of young households (Makay, 2020), one of them is the Family Housing Allowance announced in 2014 (further on: FHA)[4]. The allowance in its original form provided families that commit to having three children in total with a 10

[4]Legislated by Government Decrees No. 16/2016 and No. 17/2016 (Hungarian Government, 2016a,b)

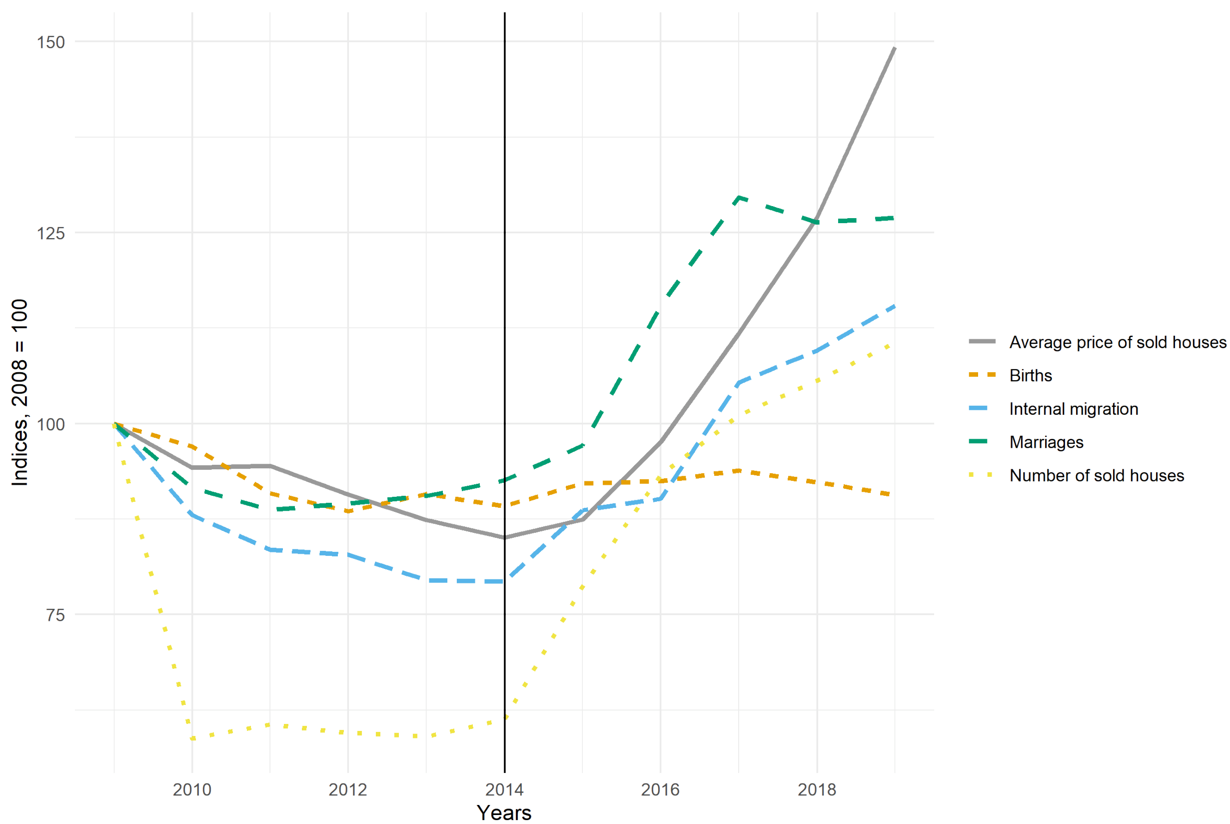
million HUF (∼30,000) lump sum for purchasing a newly constructed house that satisfies specific minimum quality requirements. At the same time, another 10 million HUF interest-subsidized mortgage was also introduced to go along with the allowance. Later, the policy was extended to purchasing owner-occupied dwellings and to families that plan to have one or two children, but the latter only with a substantially lower sum.

There were several restrictions and penalties built into the policy. Only couples where one of the parties is below 40 years of age could apply. If a family does not fulfill the requirement of the number of children they committed to (except for medical reasons), they must pay the penalty. If they have two instead of three children, they have to pay 7,400,000 HUF (∼20,000 EUR); if less than two, they have to pay back the whole 10,000,000 HUF (∼30,000 EUR), with interest at a yearly rate of five times the size of the central bank's policy rate (set at 0.9% at the time). The 'schedule' of promised children depends on the number of children the family has at the time of applying for the subsidy. If they have two children (so they only promise one extra child), the couple has four years to fulfill the commitment; for two additional children, it is eight years; for three, it is ten. Another restriction is that a dwelling purchased with the subsidy cannot be resold for ten years.

The Hungarian Government declared two objectives (Hungarian Government, 2016a; Sági et al., 2017). On the one hand, she aimed to support families in raising children and purchasing new homes. Survey evidence shows that in Hungary, house ownership is perceived as a necessary condition for raising children (Szalma and Takács, 2015). Other survey evidence by Kapitány (2016) suggests that when young adults were asked to rank several obstacles to having children, housing was named a significant but not a top issue. However, two demographic groups were found for whom housing was mentioned as a more critical aspect. One group includes those who do not plan to have a child in the short run but do in the long run, and the other group consists of those who do not want more children due to inadequate housing conditions. We can think of these groups as those the government might 'intend to treat'. The other declared aim of the legislation was to revitalize a struggling housing market with a demand push by young households, as it could not reach the pre-crisis output levels of the 2000s. The report of the Hungarian National Bank (HNB, 2019) shows that in 2007 the number of newly built dwellings stood at around 35,000, while in 2015, only at about 8,000, and did not show signs of recovering then.

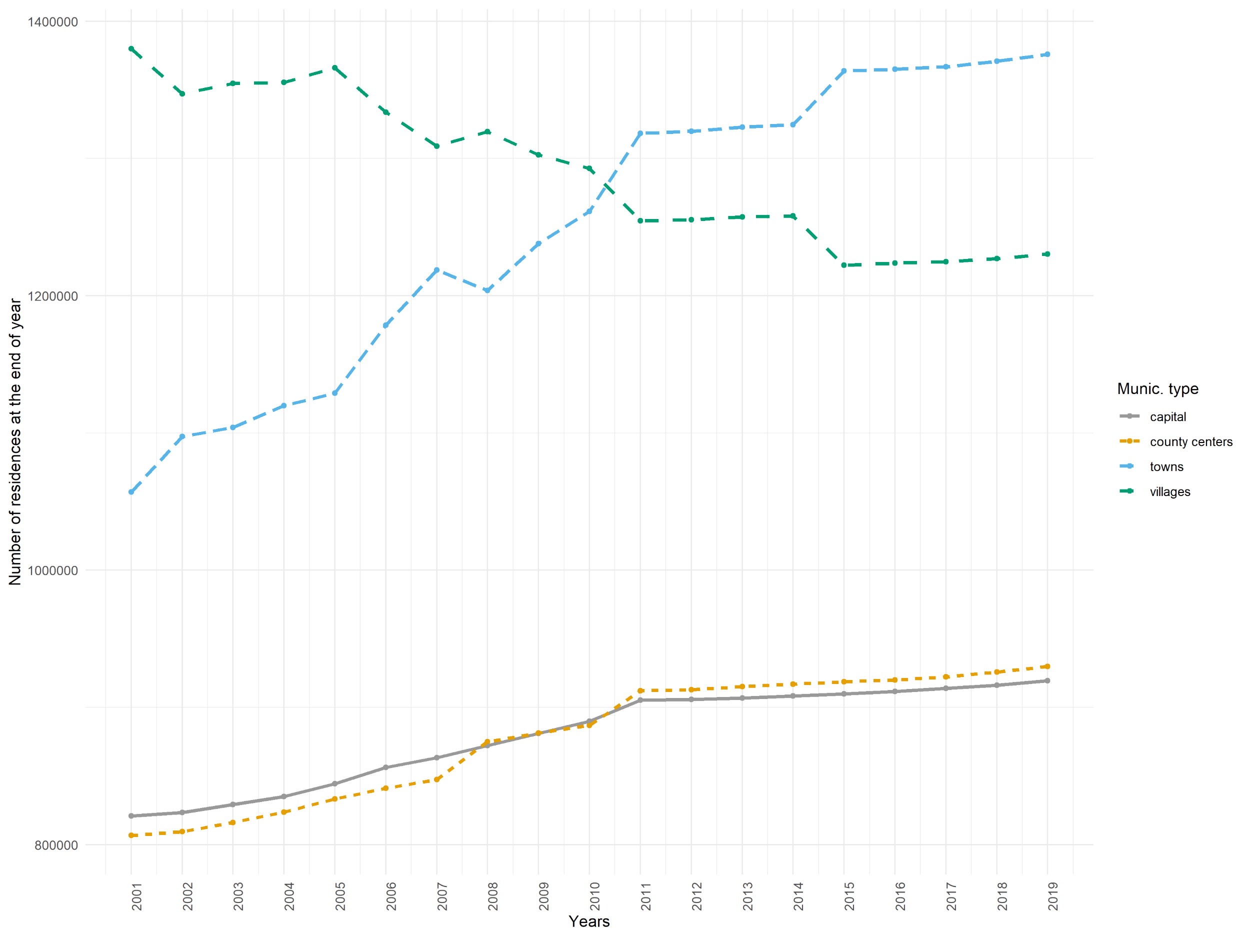
As illustrated by Figure 1, after 2014 and 2015, several macroeconomic indicators connected to the policy seem to have been perturbed. The average price and number of occupied houses sold increased dramatically, along with the number of marriages and internal migration. In contrast, the number of births seemed to experience more modest growth. Figure 2 displays the total number of houses at the end of calendar years by type of municipality. It shows that while the number of houses in rural towns seemed to increase after the policy, the rate of change for other municipality types does not seem to be substantially affected, at least in the short run.

Figure 1: Prices and number of sold houses, marriages, internal migration, and live births, 2008-2018



Note: The yearly time series are based on the publicly available settlement level data of the HCSO, accessed 16/02/2020. The vertical line indicates the announcement of the Family Housing Allowance program.

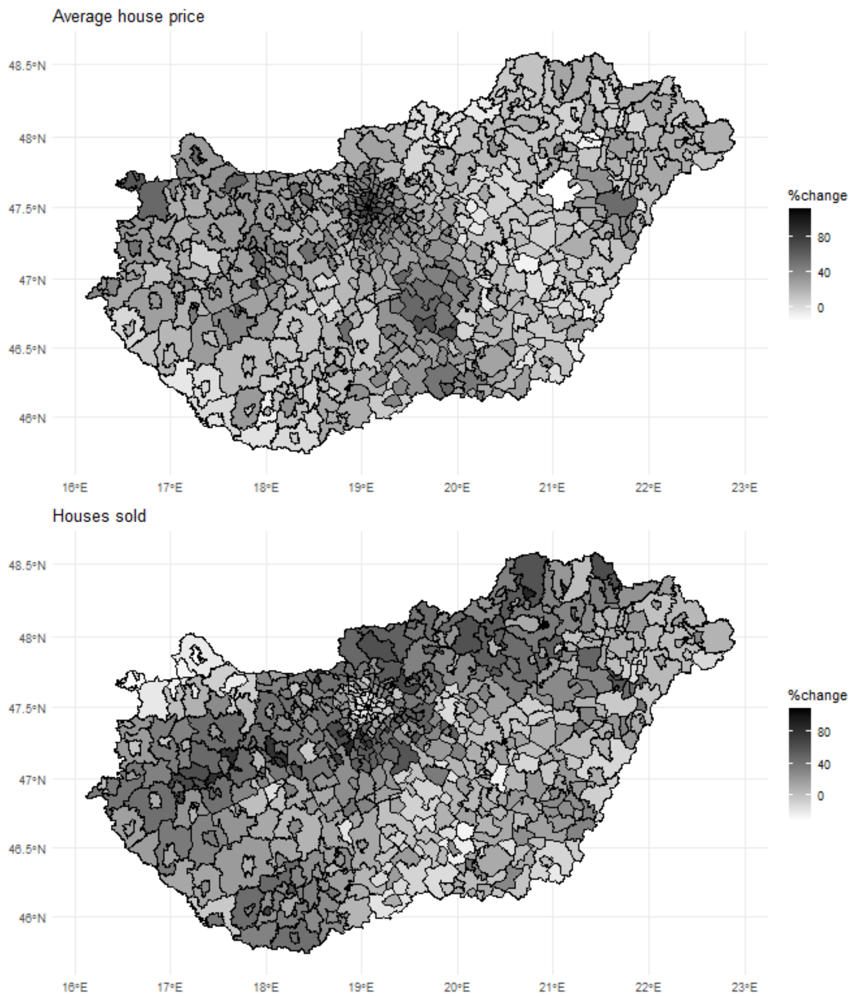
Figure 2: Total number of houses by municipality type



Note: The yearly time series are based on the publicly available settlement level data of the HCSO, accessed 16/02/2020.

There is also substantial heterogeneity within the country regarding prices and the number of houses sold, comparing the periods 2015-2019 and 2008-2014. Figure 3 shows the percentage change on the municipality level, with smaller municipalities aggregated to their next administrative unit ('járás'). Prices increased the most in Budapest and closer towns (by more than 50%, reaching levels of 80%), while at the bottom of the distribution we find the country's eastern regions. However, the change in quantities sold correlates negatively with price changes: Budapest districts experienced a decline in the number of houses sold, while in rural areas, higher numbers were sold than before, also reflected in increasing immigration to rural areas, and decreasing immigration to the center, Budapest.

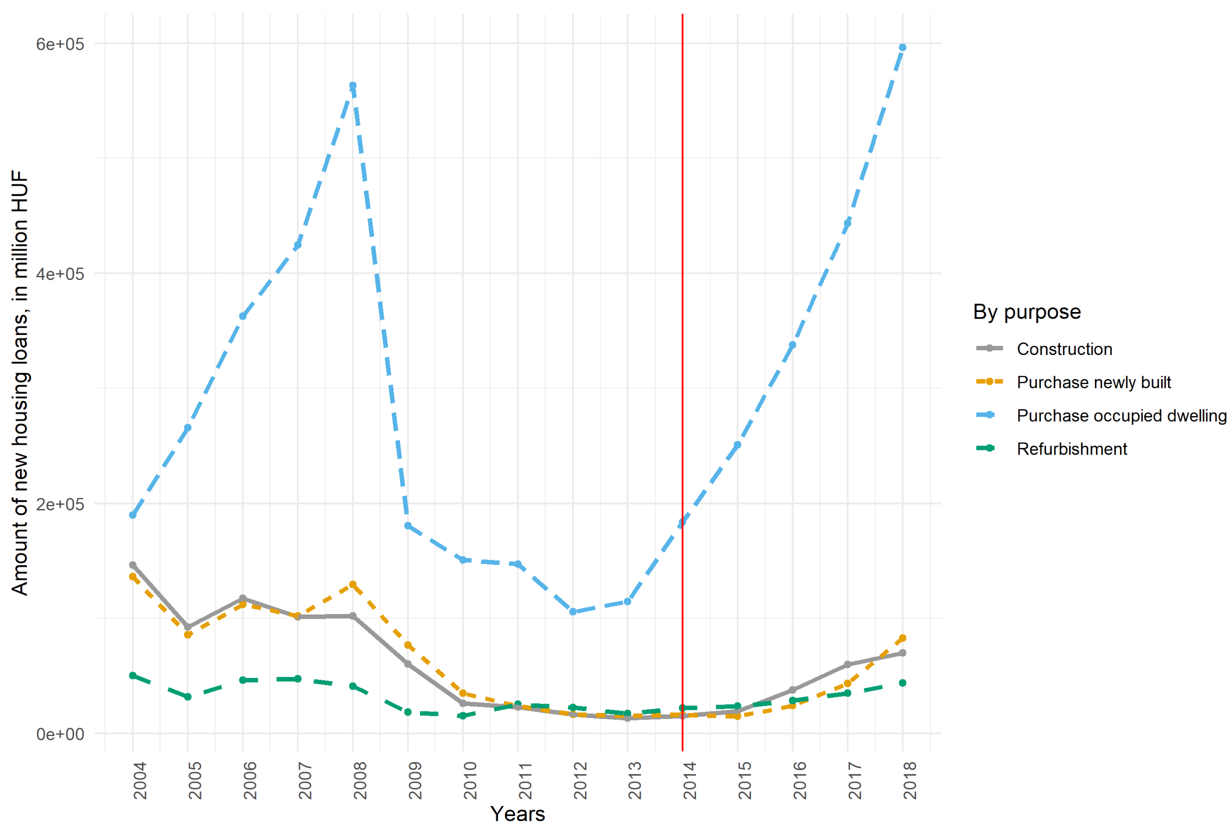
Figure 3: Change in average prices and number of houses sold in Hungary, 2015-2019 vs. 2008-2014



Note: based on the publicly available municipality-level data of the HCSO, accessed 16/02/2020. The time windows reflect the state of the housing market in Hungary before and after the introduction of the Family Housing Allowance.

The effects of the policy are far from trivial (HNB, 2021). The impact on housing demand seems substantial but not necessarily positive overall. Since 2016 close to 20% of mortgage contracts are linked to the FHA subsidy, but surging house prices restricted the accessibility of purchasing a house (HNB, 2019). That is especially true for the capital Budapest, which experienced the largest growth of house prices within the last decade in the European Union, reaching by the second quarter of 2019 around 230% of the 2009 price level. The supply side effect also manifests in the growing number of permits issued starting from 2015; the number of newly built dwellings remains at a low of 12,000 yearly, less than half of the 2007 level. Figure 4 displays the amount of newly issued housing loans, which shows that the growth in the related outstanding debt can be mainly attributed to purchasing occupied dwellings. A debate about whether the FHA policy could have affected the number of births in Hungary is also ongoing. Kapitány and Spéder (2020) suggests that the increasing number of live births fit into the regional trends of recovering fertility levels and might have happened regardless of the family policies. At the same time, Sági et al. (2017) shows indicative evidence that some of the growth could be attributed to the policy introduced.

Figure 4: The amount of new housing loan value issued in millions of HUF by purpose, 2004-2018



Note: based on the publicly available data of the Hungarian Central Statistical Office.

There are also other stylized facts to consider, which caution against overly simplistic interpretations of the data points, and show important limitations to the model this paper presents. Based on the survey published by the Hungarian National Bank HNB (2019), between 2015 and 2019, more than 40% of all house purchases in Budapest could be categorized as 'with investment purpose'. At the same time, the fraction of 'first home purchases' has fallen from around 35% to 20% of all transactions. While the FHA policy aims at buying newly constructed houses, their magnitude is small compared to the approximately 160,000 house market transactions a year, as shown by Figure A9 (even though the entire market might be affected due to substitution effects).

3 The model

3.1 Demand side: the problem of the household

The demand side of the model examines how fertility, residence size, and location choice of young households evolve over the life-cycle, with a finite number of periods. Household formation and divorce are not included in the model, the couple has already finished their education, and the female member is 25 years old. Each period, they make a forward-looking decision to have an additional child, and about the size of the house, ownership, location, and choosing between non-durable consumption and savings. They can also apply for a government subsidy representing the Hungarian government's Family Housing Allowance (FHA), providing a one-time, lump-sum subsidy in case of a house purchase with the additional commitment to having three children. Each household i is characterized by their desired number of children (ν_i), the education of the male (ed_i^M), and the education of the female (ed_i^F). Education levels are observed, but the desired number of children remains hidden, providing the source of unobserved heterogeneity in the model. However, the population distribution of desired children can be estimated, which allows for assigning households a value drawn from this distribution. Kapitány and Spéder (2020) also reports that these preferences have stayed stable during the last decades. This approach allows the model to avoid assuming an error term distribution driven by the inference strategy itself, as we can treat this distribution known. Households also face idiosyncratic uncertainty due to unemployment shocks, infertility shocks, and global uncertainty due to changing house prices.

The household's utility function $u(.)$ depends on the non-durable period consumption and relevant state variables: the number of children, the size of the residence, its location, and ownership status. Novelty in this paper's approach is that each household has its desired number of children, compared to the usual way of treating children in the utility as purely 'normal goods' such as consumption. So more than the desired number will decrease utility ceteris paribus, implying that households would not opt to have infinitely many children, even with unbounded resources. However, the housing allowance might induce households to end up with more than their desired number of children to afford better housing by taking advantage of the subsidy.

Another feature is that 'housing services' receives an intuitive meaning as the household suffers disutility from the crowdedness of the dwelling in the form of having the number of children divided by the house size in the utility function. This element provides a natural mechanism for larger families to sort into larger houses. House sizes are discretized instead of continuous housing services, and households derive extra utility from ownership, both aspects also featured in Attanasio et al. (2012).

Households suffer disutility (or gain utility) due to distance from the central location, resembling the standard monocentric city model (Duranton and Puga, 2015). It represents amenity or disamenity effects of living in the central area of the economy, an idea appearing, for instance, in Brueckner and Zenou (1999) or Combes et al. (2019). In the model, I introduce two locations representing the center and rural areas, each with a separate housing market supply, connected by the demand for housing from households concerning both locations. Wages received by the agents also differ according to the location of the household, calibrated to the Hungarian case: wages are higher in the central location but not reacting endogenously to household decisions.

I introduce two house sizes. Size 0 represents a 50 m^2 apartment (approximately 1-2 rooms), while Size 1 represents the ownership of a 100 m^2 house (around 3+ rooms). The price of housing can vary by location but is given in m^2 prices; hence the price of 50 m^2 residence is precisely half of a 100 m^2 one[5]. Table 1 shows the empirical counterparts of

[5]The ratio of the two house prices could have been set as a parameter such as Attanasio et al. (2012), here I simplify to have fewer parameters by assuming that house prices scale linearly with size

these idealized concepts based on the 2011 Census of Hungary. Looking at all residences in Hungary, around 2 million have 1-2 rooms, and around 2 million have three or more, with median sizes of 54 and 90 m^2, which types of housing are approximately represented in the model. Not being exposed to costs of owned housing is another simplification that captures the idea of imputed rents, which is found to benefit house owners substantially (Kilgarriff et al., 2019). Households can choose between renting a house and purchasing one while selecting the house size. If the residence is not owned, the household must pay rent, set at 5% of the house price corresponding to a Hungarian context (and similarly to Attanasio et al. (2012)).

Table 1: Distribution of house sizes in Hungary, 2011

No. of rooms	No. of houses	Median size (m2)	Mean size (m2)	Std. Dev.
1-2	2,135,857	54	56.93	19.41
3+	2,233,623	90	93.99	29.14

Note: based on the 2011 Census of Hungary.

I incorporate the Family Housing Allowance into the model in the following way. As the policy is intricate and requires too many new state variables, I refrain from including all details. First, I only consider the original version of the allowance with the condition of three children, as the size of the subsidy is considerably larger than for two children (10 million HUF compared to 2.6 million HUF). Second, the fertility commitments of the households are checked only at the terminal period, representing the end of fecundity, even though the actual regulation gives couples four years to have one additional child, eight years for two, and ten years for three children. This way, I avoid tracking the children's birth years, reducing computational needs significantly. Third, the dwelling purchased using the FHA subsidy cannot be sold in the model, even though selling is forbidden only for ten years; this simplifies the model's state space. Fourth, a family can only apply for the subsidy when they purchase a large house, here in the model, a 100 m^2 house. This choice represents that the policy has a minimum 60 m^2 built-in threshold regarding size[6]. And finally, only those

[6]Note, that a study of the Hungarian National Bank (HNB, 2019) finds that this hard threshold itself

households are eligible for which the age of the household (age of the female) is lower than the maximum age for which birth is possible, which is set at 40 years in the model.

The problem setup builds on the standard macroeconomic mortgage models, presented in Davis and Van Nieuwerburgh (2015). Households are modeled to be alive for a finite number of periods, making decisions from 25 until 45 years of age of the female (10 periods, or 20 years), and living an additional 20 years [7]. When they remain both in the same type of house and the same location, they are considered not to move; otherwise, they are always assumed to be able to sell or buy for the house's market price. In case they move to a residence of a different size or a different location, they suffer a moving cost as in Attanasio et al. (2012), set at 10% of the price[8], representing transportation and legal costs of the choice. Houses are expensive (prices are calibrated to reflect the property prices of Hungary), and households can use savings, their property, and mortgages to purchase them. If they do not own a house, they must rent an apartment. Outside of mortgage loans, households are only allowed to have positive savings, which they could use for consumption smoothing. The repayment schedule of mortgages is not specified, providing flexibility and inclusion of several regimes, although different institutional contexts are found to affect household behavior (Chambers et al., 2009).

3.1.1 Choice variables

Choice variables include non-durable consumption ($c_{i,t} \geq 0$) set simultaneously with the next period's savings ($S_{i,t+1} \in \mathbb{R}$), the willingness to have an additional child ($n_{i,t} \in \{0,1\}$), house ownership ($o_{i,t} \in \{0,1\}$), size ($h_{i,t} \in \{0,1\}$) and location ($l_{i,t} \in \{0,1\}$), the choice of taking out a mortgage ($m_{i,t} \in \{0,1\}$), and the choice of applying for the government subsidy Family Housing Allowance ($f_{i,t} \in \{0,1\}$).

The rules for the choices are as follows. Each household can own up to one residence. Savings can be used for consumption smoothing or mortgage repayments; however, a household can only get indebted when they take out a mortgage. This restriction is in accord

might have induced a distribution change of size among newly built apartments such that instead of the 50 m^2 apartments 60 m^2 ones have started to be constructed.

[7] About the impact of choosing finite vs. infinite number of periods, see Hedlund (2018).

[8] Compared to Attanasio et al. (2012) which uses 5% for the United Kingdom, I implement a 10% cost to reflect the considerably lower levels of real estate prices in Hungary

with evidence that historically, mortgage debt has constituted around 70% of all household debt (Piazzesi and Schneider, 2016). Following Attanasio et al. (2012), I allow households to take out a mortgage even if they choose not to move (borrow against their property), following the results of Cloyne et al. (2019), who find that house prices are the drivers of households' ability to borrow via collateral effects. Households are not allowed to have more than one mortgage contract at the same time, but they are allowed to apply for a new one after repaying the previous one. They are also forbidden to sell their house if they have a mortgage against it. They are also prohibited from selling using the FHA subsidy if they purchase it. If they are indebted in one period, they must weakly increase their position over time. Children can only be conceived until 15 years into the model, representing the end of the female fertility cycle at around 40 years. They are forbidden to apply for the subsidy past that age, which condition reflects the regulation's intentions.

3.1.2 State variables

Households are assigned at the beginning their desired number of children ($\nu_i \in \{0, 1, 2, 3\}$), and the education of male and female adults ($\mathrm{ed}_i^M \in \{0, 1\}, \mathrm{ed}_i^F \in \{0, 1\}$, 1 representing completed tertiary education). $\Omega_{i,t}$ collects the state variables for household i in period t, consisting of the following: house prices ($p_t^H(h_{i,t}, l_{i,t}), \forall h \in \mathcal{H}, l \in \mathcal{L}$), employment status ($\mathrm{e}_{i,t}^M \in \{0, 1\}, \mathrm{e}_{i,t}^F \in \{0, 1\}$), number of children ($N_{i,t} \in \{0, 1, 2, 3\}$), current house type and location ($H_{i,t} \in \mathcal{H} = \{0, 1\}, L_{i,t} \in \mathcal{L} = \{0, 1\}$), and the savings of the household ($S_{i,t} \in \mathbb{R}$), with additional states tracking the household's status status regarding mortgage ($M_{i,t} \in \{0, 1\}$), ownership status ($O_{i,t} \in \{0, 1\}$) and the government subsidy ($\mathrm{F}_{i,t} \in \{0, 1\}$ for Family Housing Allowance). Lastly, households face infertility shocks ($\iota_{i,t} \in \{0, 1\}$), which are realized only after deciding on having an additional child or not. The distribution of infertility shocks depends on the female's age and education and is considered to be known by the households.

The number of children, the house size and location, savings, mortgage, and government subsidy is determined in the previous period endogenously by the optimal choices of the households, taking the infertility shocks into account. I do not introduce uncertainty into the household's survival, examined, for instance, for the case of divorce by Fischer and

Khorunzhina (2019), or death. It is plausible that the policy affects couple formation and separation as well. However, these aspects are beyond the scope of this paper.

3.1.3 Stochastic processes

There are five stochastic processes in the model: the employment status of the male and the female adults of the households separately, infertility shocks depending on the age and education of the female, and the house prices of the central and the rural location. Unemployment shocks are realized before the choices of each period, and house prices are determined jointly with household decisions. In contrast, infertility shocks are realized after choices are made, but the distribution of infertility shocks is known to households.

The employment status of the male and the female adults follow separate, two-state Markov processes, which depend on the level of education[9]. Households form correct beliefs over the transition probabilities of employment status. These probabilities are set such that the stationary state unemployment rate corresponds to the long-run unemployment rates in Hungary conditional on education for the relevant period, which is around 9% for lower than tertiary, and 3% for tertiary educated.[10].

House prices are calculated as feasible 'pseudo-equilibrium' outcomes, which concept I introduce later in detail. Households are assumed to be price takers, and they form their expectations in a naive way over future m^2-prices such that $\mathbb{E}_t[p^H_{l,t+1}|\Omega_{i,t}] = p^H_{l,t}$, for $l \in \{0, 1\}$. This expectation formation can also be seen as households falsely believing that the time series of house prices follow a simple random walk process. Even if this belief is false within the model, there is evidence that real house price time series are indeed random walks (Holly et al., 2010). It is also worth noting that more sophisticated expectation formation could be introduced to the model, which is a possibility for a future extension.

Infertility shocks are realized after households have decided to have an additional child. At the same time, they are also aware that having a child at different points in their life-cycle implies different levels of infertility risk. I estimate these distributions conditionally on

[9] Although there could be good reasons to consider location-dependent employment transitions, a possible future improvement of the model.

[10] Based on the data of the Databank of the Institute of Economics, available as of 03/02/2021 here, and here.

female age and education[11], as the fraction of miscarriages of births added to miscarriages, for the period 2004-2014, using the complete individual-level administrative data collected by the Hungarian Central Statistical Office. Note that due to miscarriage events often not being discovered or reported, I probably underestimated this probability. Also, note that I do not consider abortion in the model or undesired pregnancies.

3.1.4 Initial and terminal conditions

Households are assigned an inheritance in the form of home ownership, drawn from the distribution estimated from the available Household Budgetary Survey of Hungary for the period 2004-2014 (introduced later in more detail), along with house size, mortgage, and location distributions at the age of 25 of the female. Only those households can have mortgages at the start that are simultaneously assigned home ownership, following the model's logic. These households also start with negative savings corresponding to the down payment for the owned house, set at 50% of the value of the home priced at the 2004-2014 levels, with a hard cap of 10 million HUF to avoid poorer households not being able to repay their debt.

Households start employed, with their education and their desired number of children fixed. The education group, along with the number of initial children in the household, are drawn from their joint distribution estimated from the 2011 Census ($\mathbb{P}^{\text{ed}^{M,F},N}$) (at female age of 25). The joint distribution captures assortative mating and selection into education tracks due to preferences, studied for instance in Adda et al. (2017). The desired number of children is drawn from population distribution $\mathbb{P}^{\nu}$, independently from the parents' educational attainment, as this information is unavailable.

The household's government subsidy uptake also impacts the terminal period savings. Households are allowed to violate the commitment to the government regarding children, but it comes with a penalty. If they decide not to respect the requirement of having the number of children they committed to, they are forced to pay back the remaining corresponding subsidy amount to the government as a lump sum at the end. If they have two instead of three children, they must pay 7,400,000 HUF ($\sim$20,000 EUR). If less than two, they have to

[11] I also estimated the distributions w.r.t. child parity, but it did not change the probabilities significantly, so I kept the simpler version.

pay 10,000,000 HUF (~30,000 EUR)[12]. Defaulting on the debt is not allowed in the model.

The terminal value $V_{i,T+1}^{\nu_i,\mathrm{ed}_i^M,\mathrm{ed}_i^F}$ captures an additional 20 years of life with the same housing conditions and the number of children they have at time T. The non-durable consumption value $\underline{c}_i$ is imputed into the post-terminal utility function, which is calculated the following way. Households are required to have a non-negative asset position by the end of the time horizon, set at 20 years. After checking the conditions for the number of children due to the government subsidy, the residual savings after calculating any penalties are checked to be non-negative. These savings are then consumed in equal shares for the following twenty years[13]. The net household income is also assumed to remain constant for future periods, and the probability of unemployment is set to 0. Specifying a terminal value is necessary from a modeling perspective as households would often sell their residence in the last period if they see no further utility in keeping it. It is equivalent to a particular type of bequest where the utility derived from the additional 20 years of owning the residence provides the bequest incentive.

3.1.5 Period utility function

Households derive instantaneous utility from non-durable consumption, the number of children, the crowdedness of the house, location, and home ownership in the following way:

$$u_{\mathrm{ed}^F}(c_{i,t}, N_{i,t}, H_{i,t}, L_{i,t}, O_{i,t}) =$$
$$\frac{(c_{i,t}-1)^{1-\gamma_{\mathrm{ed}^F}}-1}{1-\gamma_{\mathrm{ed}^F}} \exp\Big(- w^N_{\mathrm{ed}^F}(N_{i,t}-\nu_i)^2 - w^H \frac{N_{i,t}+1}{I[H_{i,t}>0]+1} - w^L L^2_{i,t} + w^O I[O_{i,t}>0]\Big)$$

where $I[.]$ denotes the indicator function. The utility function is multiplicative, reflecting complementarity between the components of the function. As mentioned earlier, ν_i denotes the desired number of children, which is the source of unobserved heterogeneity, responsible for observationally identical households ending up with different choices. However, I censor the distribution to include only 0-3 preferred children, with values above three incorporated

[12] I abstract away from the penalty interest paid, as it does not change the magnitudes of the penalty even if they might be significant, but would introduce unnecessary complexity in the computations.

[13] There could be a future point of improvement by setting consumption to be based on the consumption Euler equation. In this version, I stayed with the simpler setup

into the highest category. I also let agents have three children at maximum. Notice that households with lower and higher educated females are allowed to have different risk aversion parameters (γ_{ed^F}) and different preferences for children ($w^N_{\text{ed}^F}$). The latter concept is represented as suffering disutility from being further away from the desired number. Allowing for education-dependent parameters captures the previous self-selection into different education and career tracks according to preferences about children and consumption, as indicated by Adda et al. (2017). Consumption is constrained from below by 1 with a price of p^c, which denotes the living costs corresponding to the subsistence level based on the calculations of the Hungarian Central Statistical Office (HCSO, 2016)[14]. House size $H_{i,t}$ is coded as a binary variable, representing 50 and 100 m^2 apartment sizes as 0 and 1. However, the number of children also affects the crowdedness of the house (third term in the utility function), with disutility calculated as the number of children over the house size. House size is modeled such that the small one is half the size of the large apartment, be it owned or rented[15]. The household also derives utility/disutility from being further away from the city center, representing the city center amenities. Ownership status also provides positive utility, as it is generally assumed and found in the literature (Davis and Van Nieuwerburgh, 2015).

3.1.6 Budget constraint

$$p^c\Big(1+\frac{N}{3}\Big)c_{i,t} + S_{i,t+1} + \kappa(n_{i,t}, N_{i,t}) + p^H_t(h_{i,t}, l_{i,t}) + \mu(H_{i,t}, L_{i,t}, h_{i,t}, l_{i,t}, \mathrm{F}_{i,t}, M_{i,t}) +$$
$$\rho(H_{i,t}, L_{i,t}) \leq NW(W^M_{i,t}, W^F_{i,t}) + (1+ir)S_{i,t} + \mathrm{FHA}(f_{i,t}) + p^H_t(H_{i,t}, L_{i,t}, O_{i,t})$$

The budget constraint builds on the baseline setup of macroeconomic models with mortgage decisions (Davis and Van Nieuwerburgh, 2015). The first term is the value of the total household consumption $p^c\Big(1+\frac{N}{3}\Big)c_{i,t}$, with a child's consumption corresponding to about one-third of an adult couple, per the estimates of the Hungarian Central Statistical Office (HCSO, 2016). This term also introduces the flow cost of children in terms of consumption such that higher consumption of adults in the household also induces higher spending on

[14] I do not normalize by this value to impute nominal prices more easily during the model development.
[15] Differently from Attanasio et al. (2012), renting larger houses is allowed for the households.

children in a linear way. Therefore, we implicitly assume a structure of preferences regarding child quality, which is omitted from this model (discussed in Sommer, 2016). As discussed earlier, the next period's savings are denoted by $S_{i,t+1}$. The function $\kappa(n_{i,t}, N_{i,t})$ denotes the costs and benefits associated with children in the household. It includes child benefits, however, also the out-of-pocket spending for giving birth in Hungary, which might be substantial, even if not reported officially[16]. The function $p_t^H(h_{i,t}, l_{i,t})$ assigns a price to a house with house type $h_{i,t}$ and location $l_{i,t}$, while $\rho(H_{i,t}, L_{i,t}, O_{i,t})$ gives the rental cost or house service costs associated with the house type at t (the latter assumed to be 0 for now). Finally, the function $\mu(.)$ introduces moving costs, which gives incentives for agents to remain sedentary (as often used in the literature, per Davis and Van Nieuwerburgh, 2015). The moving costs are set up such that houses under mortgage commitment cannot be sold while being 10% of the house price in case of selling.

The revenue side of the budget is the following. The net income of the household, $NW(.)$, is the function of the gross incomes of the male and female in the household[17]. Unemployment spells are set to one year, during which the unemployed person receives lower wages[18]. In the case of a new child, the woman's salary is accounted for as two-thirds approximating the amount of the benefits maternity policies provide in Hungary (Makay, 2020).

Gross income is given by a Mincerian reduced form equation of education, employment status, location choice, and experience (here measured by the age of the household). Women face less experience due to their number of children to address the opportunity cost of child birth, and each child counting as two years less experience[19]. The equations are the following, where $e_{i,t} \in \{0, 1\}$ represents employment, and $W_{\min}$ denotes the amount representing

[16] Based on the blog of the Hungarian National Bank, in Hungarian: https://novekedes.hu/elemzesek/ne-kamuzzunk-sulyos-szazezrekbe-kerul-magyarorszagon-az-ingyenes-szules

[17] Since 2011, the number of children plays a more significant role in personal income taxation, however for simplicity in the model I set taxes at 42%, consisting of a 15% personal income tax, and a 27% social contributions, reflecting the taxation of the era.

[18] Calibrated to the minimum wage as of 2011 at around 80,000 HUF a month

[19] Following maternity benefits design of Hungary, Makay (2020)

unemployment benefit:

$$W^M_{i,t} = \mathrm{e}_{i,t} \exp(\beta^{W,M}_0 + \beta^{W,M}_1 X_{i,t} + \beta^{W,M}_2 X^2_{i,t} + \beta^{W,M}_3 \mathrm{educ}^M_i + \beta^{W,M}_4 L_{i,t}) +$$
$$(1 - \mathrm{e}_{i,t}) W_{\min}$$
$$W^F_{i,t} = \mathrm{e}_{i,t} \exp(\beta^{W,F}_0 + \beta^{W,F}_1 \min\{X_{i,t} - 2N_{i,t}, 0\} + \beta^{W,F}_2 \min\{X_{i,t} - 2N_{i,t}, 0\}^2 +$$
$$\beta^{W,F}_3 \mathrm{educ}^F_i + \beta^{W,F}_4 L_{i,t}) + (1 - \mathrm{e}_{i,t}) W_{\min}$$

The parameters are estimated by regressing the log of gross wages on experience, education, and location, separately for the two genders[20]. Note that these parameters should not be interpreted as causal effects of the variables; they should only be interpreted as coefficients of the best linear prediction.

3.1.7 Recursive form

The household problem can be summarized in a recursive form in the following way:

$$V^{\nu_i, \mathrm{ed}^M_i, \mathrm{ed}^F_i}_{i,t}(\Omega_{i,t}) = \max_{\substack{c_{it} \geq 1, n_{i,t} \in \{0,1\}, \\ h_{i,t} \in \mathcal{H}, l_{i,t} \in \mathcal{L}, o_{i,t} \in \mathcal{O} \\ m_{i,t} \in \{0,1\}, f_{i,t} \in \mathcal{F}_{i,t}, \\ S_{i,t+1} \geq S_{i,t} I[S_{i,t} < 0] + \\ -m_{i,t}(1-\delta) p^H(h_{i,t}, l_{i,t}) I[S_{i,t} \geq 0]}} u_{\mathrm{ed}^F}(X_{i,t}) + \beta \mathbb{E}_{\sigma_{i,t+1}}[V^{\nu_i, \mathrm{ed}^M_i, \mathrm{ed}^F_i}_{i,t+1}(\Omega_{i,t+1}) | \Omega_{i,t}]$$

$$X_{i,t} = (c_{i,t}, N_{i,t}, H_{i,t}, L_{i,t}, O_{i,t})$$

$$\Omega_{i,t} = (\sigma_{i,t}, H_{i,t}, L_{i,t}, O_{i,t}, S_{i,t}, M_{i,t}, \mathrm{F}_{i,t})$$

subj. to:

[20] I omitted here the variance parameter of the log-normal distribution, which would be the appropriate form for expected value.

Budget constraint:

$$p^c\Big(1+\frac{N}{3}\Big)c_{i,t}+S_{i,t+1}+\kappa(n_{i,t},N_{i,t})+p_t^H(h_{i,t},l_{i,t})+\mu(H_{i,t},L_{i,t},h_{i,t},l_{i,t},\mathrm{F}_{i,t},M_{i,t})+$$
$$\rho(H_{i,t},L_{i,t})\leq NW(W_{i,t}^M,W_{i,t}^F)+(1+ir)S_{i,t}+\mathrm{FHA}(f_{i,t})+p_t^H(H_{i,t},L_{i,t})$$

State transitions:

$$N_{i,t+1}=N_{i,t}+n_{i,t}\cdot\iota_{i,t},L_{i,t+1}=l_{i,t},H_{i,t+1}=h_{i,t},F_{i,t+1}=F_{i,t}+f_{i,t}$$

$$M_{i,t+1}=\begin{cases}0, & \text{if } M_{i,t}=1 \text{ and } S_{i,t+1}\geq 0\\ M_{i,t}+m_{i,t}, & \text{otherwise}\end{cases}$$

Initial conditions:

$$\nu_i\sim\mathbb{P}^{\mathrm{nu}},\iota_{i,t}\sim\mathbb{P}_t^{\mathrm{ed}_i^F},$$

$$(\mathrm{ed}_i^M,\mathrm{ed}_i^F,N_{i,0})\sim\mathbb{P}^{\mathrm{ed}^{M,F},N},(\mathrm{e}_{i,0}^M,\mathrm{e}_{i,0}^F)=(1,1),$$

$$H_{i,0}\sim\mathbb{P}^H,L_{i,0}\sim\mathbb{P}^L,O_{i,0}\sim\mathbb{P}^O,M_{i,0}\sim\mathbb{P}^M$$

$$\textit{Terminal value: } V_{i,T+1}^{\nu_i,\mathrm{ed}_i^M,\mathrm{ed}_i^F}(\Omega_{i,T+1})=\sum_{s=0}^{19}\beta^s u_{\mathrm{ed}^F}(\underline{\mathrm{c}}_i,N_{i,T+1},H_{i,T+1},L_{i,T+1})$$

where i denotes a household, t denotes periods which at this point correspond to years of age for the female adult of the household, $\sigma_{i,t}$ collects the random states of employment, house prices, and the number of children, that the agents take expectation over. $u(.)$ denotes the utility or period utility function, β denotes the discount factor. Interest rate (ir) is set as a baseline at 5% a year[21] reflecting the 2013-2019 mortgage interest rates of Hungary (HNB, 2019), and $\beta=\frac{1}{1+ir}$ is set accordingly. The down payment ratio (δ) is also fixed, as a baseline at 0.5[22]. Using fixed interest rates implicitly assumes fixed-rate mortgages (vs. adjustable-rate mortgages). This type of mortgage contract has been the most prominent option in Hungary in the last decade (HNB, 2019), and it constitutes the majority in the U.S. as well (Piazzesi and Schneider, 2016).

[21] In case of biannual periods, it is set as $ir=(1.05)^2-1$

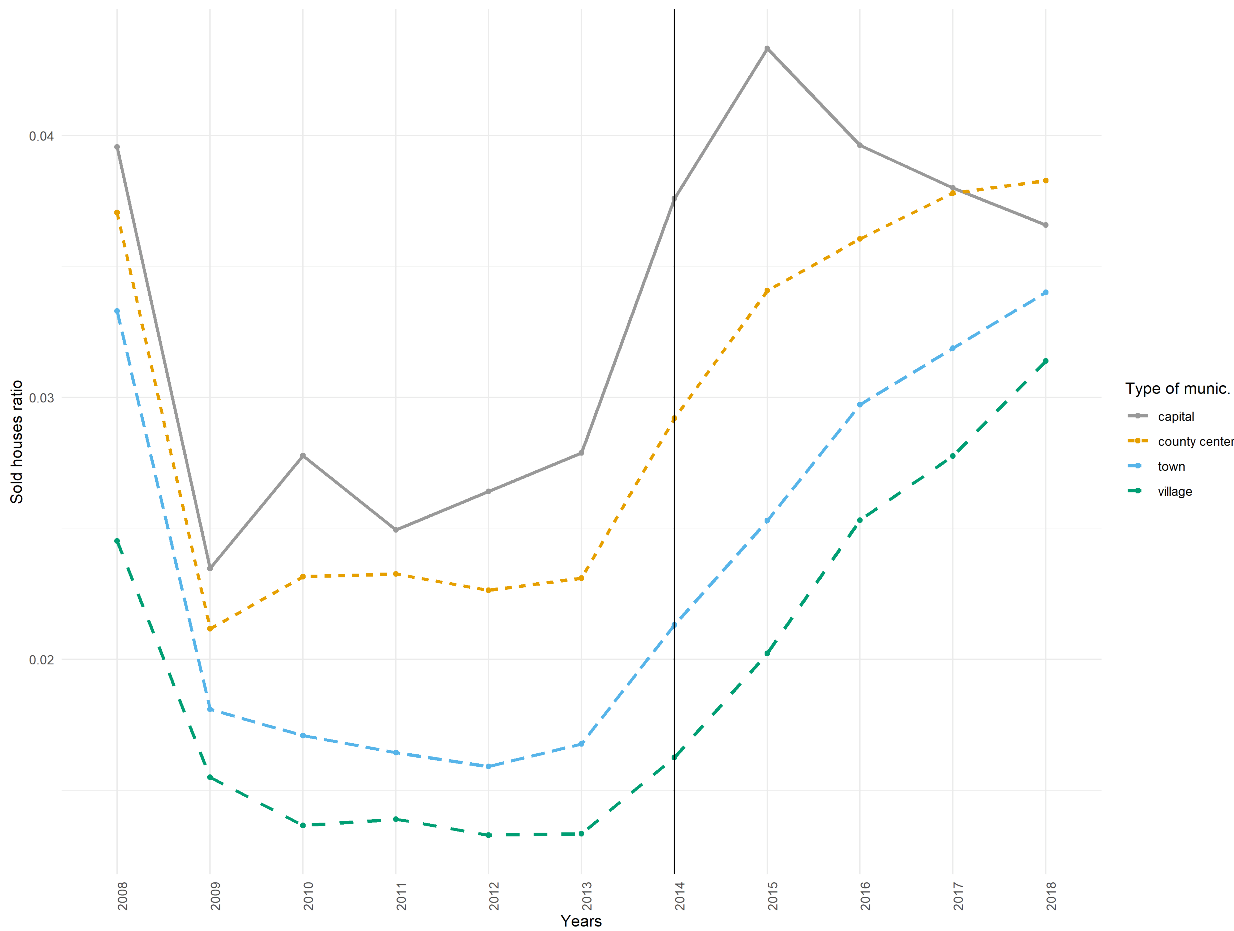
[22] According to the calculations of the website Bankmonitor.hu, specializing in mortgage loans, banks typically required a 0.4-0.5 down payment ratio in Hungary. Source in Hungarian:, accessed 29/07/2020.

3.2 House supply

House supply introduces several complexities into the analysis analytically and computationally. Hence, many in the literature treat house prices as exogenous to the household problem (Duranton and Puga, 2015). The price of housing is composed of two elements: the price of the land and the structure itself. Piazzesi and Schneider (2016) documents that the overwhelming majority of the historical variation of house prices can be attributed to land prices. It is easy to see that rising land prices can result from a sudden increase in demand, which an inelastic supply cannot follow fast enough. In my paper, I incorporate this phenomenon, which requires some level of equilibrium responses of prices. There are instances in the literature where supply is explicitly modeled, such as Glaeser et al. (2008); however, their model would not be feasible in this setting.

In my model, the supply of housing has three components. First, there is a constant per period available m^2 space for newly built houses, governed by calibration parameters for each location denoted as (α^0, α^1), representing a 'fixed stream of housing services' (Piazzesi and Schneider, 2016) often used in the literature. The revenue generated by selling these houses are not contributing to the household budgets, and I do not consider this income in the model. The available external supply of housing measured in m^2 is then calculated the following way for location l: $H_l^S = \alpha^l \cdot |\mathcal{I}| \cdot 50$ where $|\mathcal{I}|$ denotes the number of households in a cohort, in the model. So the parameter captures the relation between the amount of newly available space and the number of new households entering the housing market on the demand size. This component has the most relevance in this policy context as the Family Housing Allowance originally aimed at purchasing newly constructed dwellings. In this version of the model, this component is not reacting to price changes corresponding to a fully inelastic housing supply; experimenting with additional elements to the supply side could provide valuable model extensions. However, as Figure 5 shows, the number of real estate transactions is less than 5% of the total available houses for all municipality types, suggesting that even with sharply increasing house prices, an overwhelming fraction of owners do not sell their property. Still, at the same time, as we have seen earlier, house transactions dominantly occur in the domain of owner-occupied properties.

Figure 5: Fraction of houses sold of the total, by type of municipality, 2007-2018



Note: the author's calculations, based on the publicly available data of the Hungarian Central Statistical Office.

Second, I introduce a set of 'old' households starting with homeownership and maximizing expected utility similarly to the young households in a simplified model: without the ability to have children or to take up a mortgage or the government benefit. They react to price changes and present a finite stock of housing available on the housing market, responding to changing prices and decreasing as we move forward in time. However, in other model applications, this part of the supply could play a significantly more important role. And third, young households also constitute part of the housing supply if they change their residence. It is worth mentioning that I abstract away from several aspects which could play a role, such as the quality of the apartments, optimal portfolio decisions, AirBnB, and, most crucially, speculative or investment-focused house purchases are not considered here. These are all aspects that might be worth investigating further in the future.

3.3 'Pseudo-equilibrium' in the housing market

House prices are present in the model as state variables, which must have a small number of discrete levels due to constraints of feasibility. That setup does not allow for an actual temporary equilibrium price discussed in Piazzesi and Schneider (2016), which would clear the housing market for each location in each period. Instead, I employ the following algorithm that enables period-by-period 'pseudo-equilibrium' prices and allocations:

1. Fix period t
2. For each combination of possible location-m^2-prices collected into vector $p^H \in \mathbb{R}^2_+$, we calculate for each household i and for each location l the net demand for total space, denoted by $h^D_{i,l}(p^H)$
3. For each combination of location-m^2-prices, we also calculate the external net supply for each location l as a sum of fixed external stream of housing, added to the housing sold by the old households, denoted jointly by $h^S_{j,l}(p^H)$, j indicating different sources of supply
4. We sum up the net demand for space of the households, and the net supply from different sources, for each location
5. We calculate the squared difference of the total net demand from the total net supply and select the m^2-price yielding the least distance as the 'pseudo-equilibrium' price

So the temporary 'pseudo-equilibrium' price vectors for the housing markets can be defined in the following way:

$$(p^H)^* \in \operatorname{argmin}_{p^H \in \mathbb{R}^2_+} \sum_{l \in \{0,1\}} \omega^l \left(\sum_{i \in \mathcal{I}} h^D_{i,l}(p^H) - \sum_{j \in \mathcal{J}} h^S_{j,l}(p^H) \right)^2$$

where $p^H \in \mathbb{R}^2_+$ denotes the m^2-price vectors, $h^D_{i,l}(p^H)$ denotes the net housing space demand of household i in location l given the m^2-price vector, and $h^S_{j,l}(p^H)$ denotes the net housing space supply provided by either an 'old' household or provided externally. The ω^l parameters govern how much weight we put on each location's deviation from its market clearing, which allows for asymmetric approximate market clearing conditions for different housing markets.

This algorithm would yield a price vector for each time period and location for the housing market, so it would close the gap between demand and supply as much as the discretization allows. If the temporary equilibrium price vectors exist, this mechanism with sufficiently fine price grids will find them. However, uniqueness is generally not guaranteed (Duranton and Puga, 2015), which also stands for this algorithm.

3.4 Model solution

The model can be solved by backward induction, yielding a unique optimum due to the strict concavity of the utility function in non-durable consumption. First, I solve the model for young and old households, generating value and policy functions. In this model, prices are in temporary 'pseudo-equilibrium', meaning that they are allowed to react to the exogenously given inelastic housing supply, interacting with the housing demand of young households following its life-cycle optimization program. Therefore we must simulate the entire histories of all housing markets jointly over the life-cycle of the agents, somewhat similarly to how it is treated in the literature when exogenous price processes are separately estimated, such as Attanasio et al. (2012).

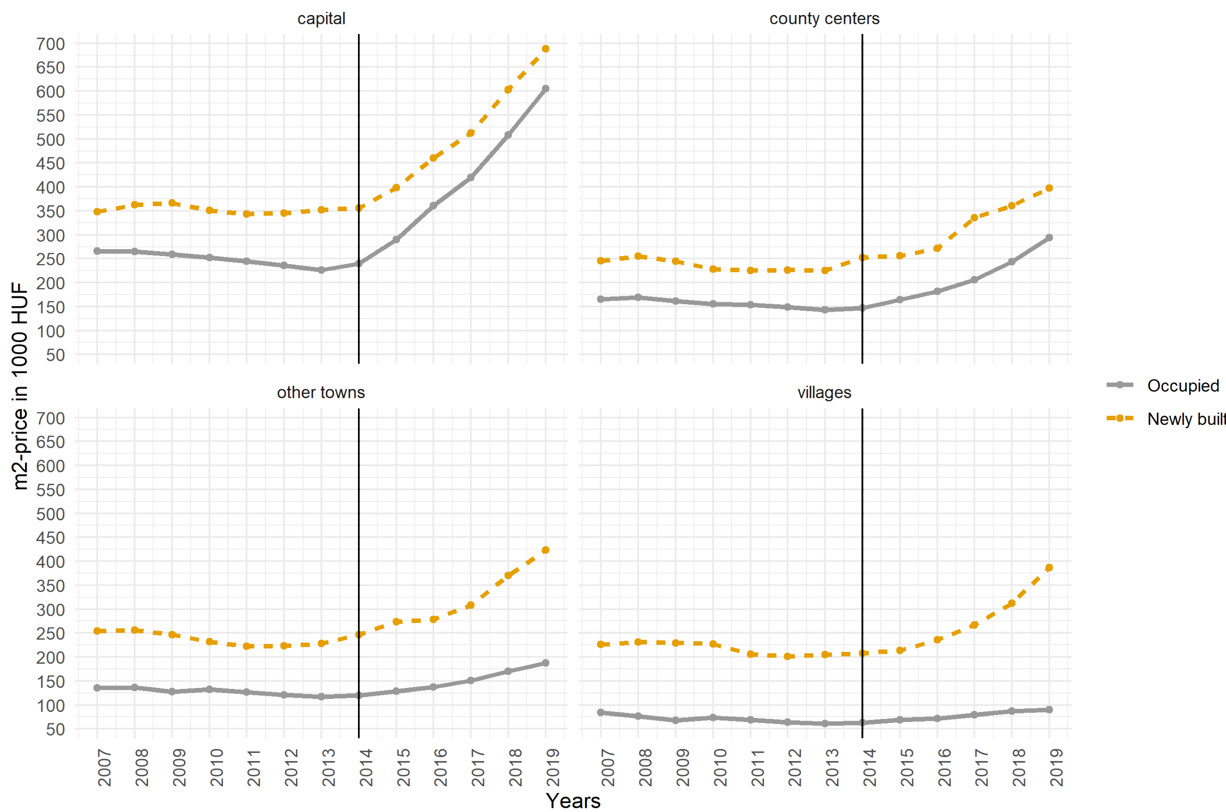
In the simulation, I include two cohorts of 150 'young' households (called Cohort-0 and Cohort-1) and 150 'old' households that act as price-sensitive supply. Cohort-0 households start at the beginning of the historical time of the simulation at age 25 and make optimal decisions according to the prescriptions of their policy functions in a price-taking manner. Cohort-1 households enter historic time two periods later, also at age 25, and make optimal decisions. However, they are lagging in their life-cycle. This modeling choice allows for distinguishing between the effects of the policy on the first impacted cohort vs. future generations who already have to face the price impacts of the policy at an earlier stage in their life. It is the representation of the assessment of the Hungarian National Bank's comment on how the increasing house prices (if indeed the model generates that dynamic) might undermine the accessibility of housing for younger generations.

The m^2-price grids include six-six possibilities (Table 2) for each of the central and the rural locations, based on the historic price dynamics displayed in Figure 6, in the Hungarian context (HNB, 2019). We can see that for our estimation period, the central location (capital) prices for newly built dwellings are around 350,000 HUF, while the other locations are approximately 250,000 HUF. As I describe later, I use these m^2-prices to estimate the household demand parameters.

Table 2: Possible m^2-prices of houses

Central location	200	300	400	500	600	700
Rural location	50	100	150	200	250	300

Figure 6: Yearly average m^2-prices by type of municipality and age of the house, 2007-2019



Note: based on the house price estimates of the Hungarian Central Statistical Office.

The savings grids differ for each education group, which allows for representing the different credit limits given by their different earning powers. I could also set the savings grids finer for the lower educated group. Each grid contains around 20 grid points. As I use the biannual model version instead of the annual one, the crudity of the savings grids plays an even less important role. As having mortgages could potentially result in significant indebtedness, it is vital to have a wide range of grid points for the savings accounts of households. Households are bound to choose a savings level from the grid point.

4 Estimation

I combine Hungarian survey and administrative datasets to estimate the model parameters using the 2004-2014 period when real estate prices did not experience significant turbulences.[23] I employ the simulated method of moments to estimate the preference parameters in the utility function. At the same time, I use the reduced-form Mincerian regressions for the wage parameters, which I treat simply as projection coefficients without any causal interpretation. I also estimate the probability distributions concerning the initial values of the state variables using the available information about households with cohabiting adults where females are 25-26 years old. The supply parameters (α) are then calibrated in a second step. Given the utility function parameter estimates, I simulate the model with endogenous prices and without the subsidy using different (α^0, α^1) supply parameters. Then I select the ones that produce the smallest squared deviation from the (350, 250) prices for the central and rural locations, respectively, that could be considered closest to the equilibrium prices for the 2004-2014 period. Figure 6 shows the average m^2-price levels for newly built and older houses as a reference point. We can see that the target prices for the second step are slightly higher for the central (Budapest, capital) location than their actual level. However, the price grids could not be extended further due to computational feasibility.

4.1 Data

The primary data source of the exercise is the Hungarian Household Budget Survey[24] , which is a yearly survey of private households, representative at the country level. The Hungarian Central Statistical Office collects detailed information on the population's consumption, income, housing, and several other demographic features, which is then used to calculate the product weights of the consumer price index. The information also contributes to national account calculations. The structure of the surveys has changed over the relevant period of 2004-2018, which required maintaining simple definitions of the variables that could map

[23] I am grateful for the help of the Databank at the Institute of Economics for providing access to the datasets used in this paper.

[24] In Hungarian: "Háztartási Költségvetési és Életkörülmény Adatfelvétel" abbreviated as HKÉF, the English description is available at
http://www.ksh.hu/apps/meta.objektum?p_lang=EN&p_menu_id=110&p_ot_id=100&p_obj_id=AEAA

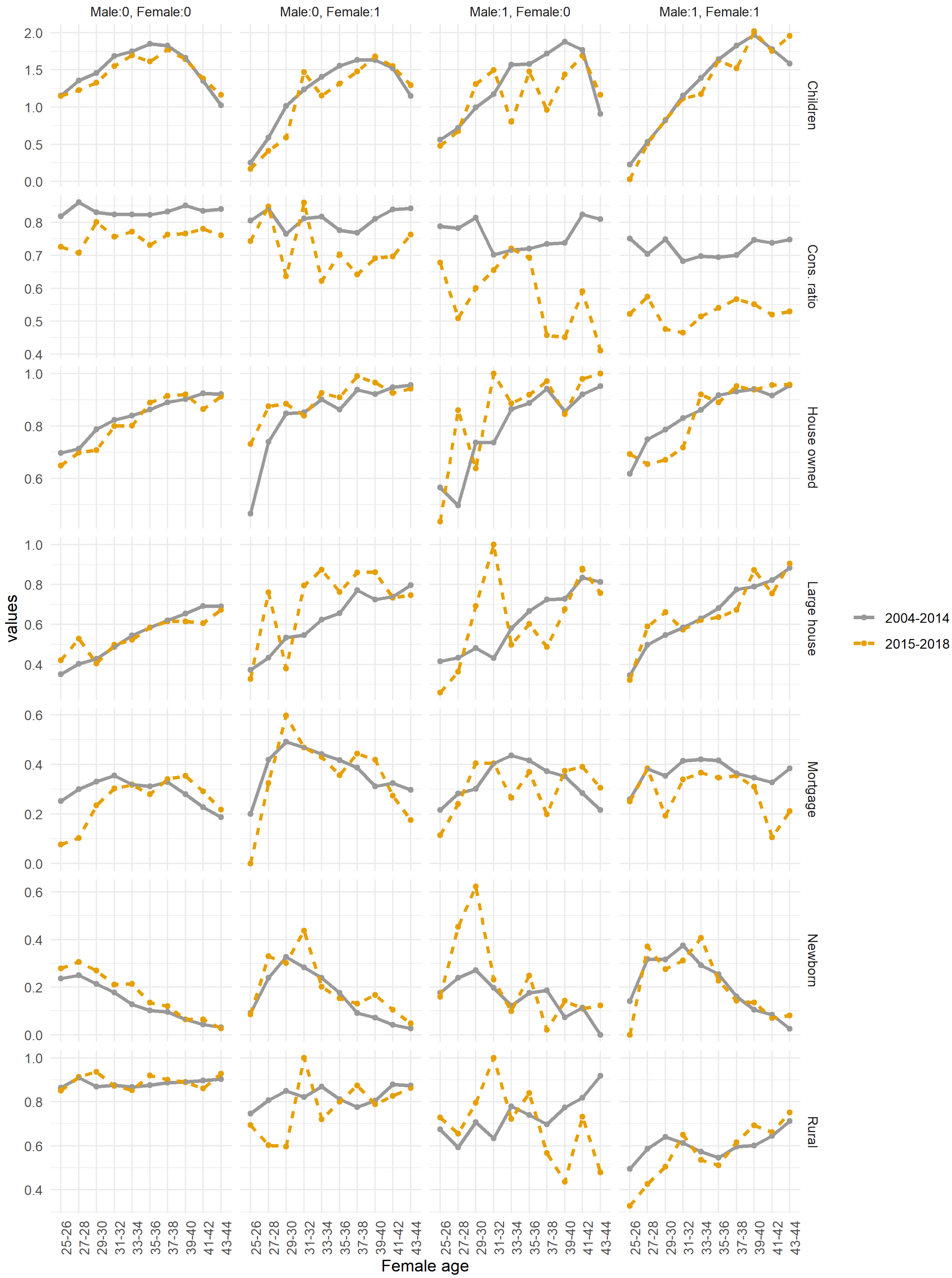
into the model. Since 2010, the survey has not included detailed nominal household income information. Therefore I used the Hungarian National Wage Survey data to impute expected net household wages conditional on the calendar year, age, gender, education, administrative region, and the type of municipality.

For each survey of 2004-2018, I kept in the sample those households which included two adults identified as male and female members of a cohabiting couple, not necessarily in marriage. Then I created the following household-level variables for the estimation of the model: consumption ratio (consumption as a fraction of net household wage income), house size category where houses with three or more rooms are categorized as large, location category based on residing in the capital (Budapest) as a proxy for the central location in the model, homeownership, mortgage, newborn children (children less than one) and the total number of children (children family status and less than 19 years of age). Finally, I pooled the pre-policy period of 2004-2014 and the post-policy period of 2015-2018 to increase the number of observations, as each survey contained only a small number of households after conditioning on education and female age.

A major deficiency of the dataset is that it was impossible to reconstruct the households' net savings position; hence, moments connected to this information cannot be used in the estimation. Another potential problem is the selection to become a couple, which is not addressed by the model or the sampling. The latter issue appears both regarding calendar time with changing family structures over time and due to the actual effect of the policy on family formation. Addressing this selection problem could be a valuable addition to the present model.

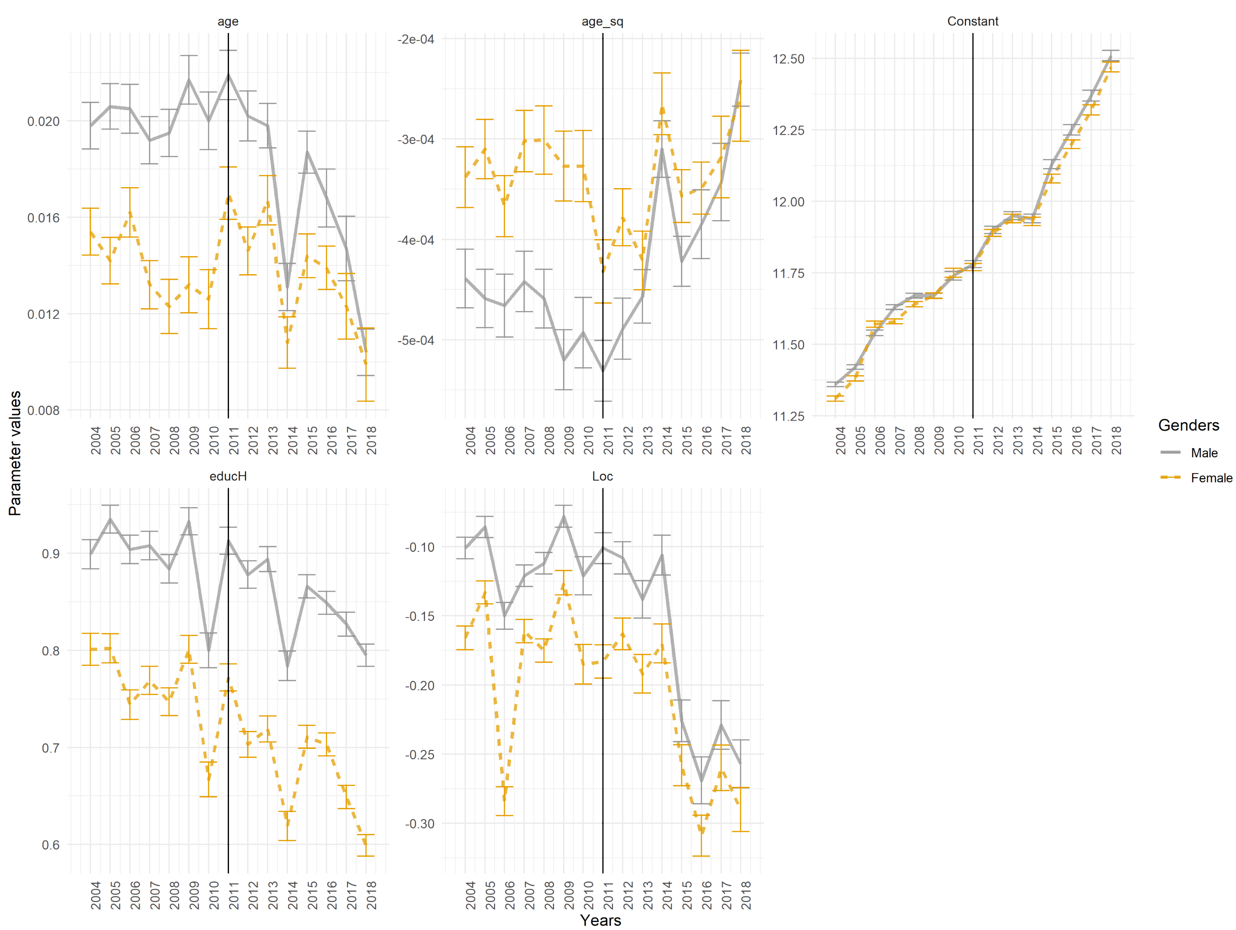
The following Figure 7 shows, by education categories and periods, the averages of the main choice and state variables as a function of the age of the female. We can immediately notice the greater variance in the time series for the shorter period due to the smaller time window. Otherwise, we can see that, on average, most variables, such as the total number of children, evolve over the life-cycle more or less similarly in both periods. The consumption ratio seems lower, which could be an unfortunate technical consequence of the wage imputation. The evolution of homeownership also appears to be similar, starting from a high level. However, we can notice a slight uptake in the ratio of larger houses in the latter period for younger households. Another thing to note is that at the older age of females, children start to leave the household resulting in a decrease in the number of children at the end, which is impossible in the model (hence I use the life-cycle maximum of the average, instead of the terminal value).

Figure 7: Average values for the main choice and state variables, 2004-2014 vs. 2015-2018



For the parameters in the Mincerian wage regressions of the model, I used the data from the Hungarian Wage Survey, which contains demographic and salary information about the labor force of firms with more than five employees. I regressed gross monthly salaries on a constant, age, age squared, and dummies for higher education and rural location, separately for each gender and year. Figure 8 shows the point estimates with their 95% confidence intervals. We can see a substantial variance in the estimates of some parameters, such as age, throughout the periods. I use the 2011 estimates as parameter inputs for the model simulations.

Figure 8: Parameter estimates for the wage equation with 95% CI



Lastly, I estimated the distributions for the initial conditions of the state variables in the following way. I used the 2011 Census data of Hungary to estimate the joint probability distribution of households where the female's age is 25-26 by male education, female education, and the number of children. For the other state variables (home ownership, large house, location, mortgage), I used the univariate distributions estimated from the Household Budget Survey, as they would have been over-saturated for a multivariate estimation. The Census data held no information on mortgages.[25] Table 3 presents the household counts and the initial moments for households by education group and period on which the imputed distributions are based.

Table 3: Averages of initial state variables by education and period, Household Budget Survey, 2004-2018

Female age	Male high educ.	Female high educ.	Period	Household count	Large house	Owned house	Rural	Mortgage	Children
25-26	0	0	2004-2014	661	0.35	0.70	0.86	0.25	1.15
25-26	0	0	2015-2018	110	0.42	0.65	0.85	0.08	1.15
25-26	0	1	2004-2014	119	0.37	0.47	0.75	0.20	0.25
25-26	0	1	2015-2018	6	0.33	0.73	0.69	0.00	0.17
25-26	1	0	2004-2014	56	0.42	0.57	0.68	0.22	0.56
25-26	1	0	2015-2018	15	0.26	0.44	0.73	0.11	0.48
25-26	1	1	2004-2014	138	0.34	0.62	0.49	0.26	0.23
25-26	1	1	2015-2018	16	0.32	0.69	0.33	0.25	0.03

Note: the author's calculation, based on the Hungarian Household Budget Survey, 2004-2018.

In the model, the unobserved heterogeneity originates from the latent desired number of children, for which the distribution is estimated in the demographic literature (Kapitány and Spéder, 2015). I display this distribution in Table 4.

[25]It is possible, however, to use the Census for a joint estimation regarding the other variables besides having a mortgage, although for some cells, the number of observations would be low even for the entire population. I decided against this option because according to the 2011 Census, around 80% of the households with 25-26-year-old females live in a house owned by them, which seems overestimated compared to the 46-70% ratios found in the Household Budget Survey. One possible explanation is that cohabiting couples might not appear as such in the census because, officially, many of them could have their permanent residence with their parents. So some cohabiting couples who live in a rented apartment might be accounted for as 'children' in the Census.

Table 4: Estimated distribution of the desired number of children

k	0	1	2	3
$\mathbb{P}^{\nu}(k)$	0.02	0.12	0.65	0.21

Note: based on (Kapitány and Spéder, 2015).

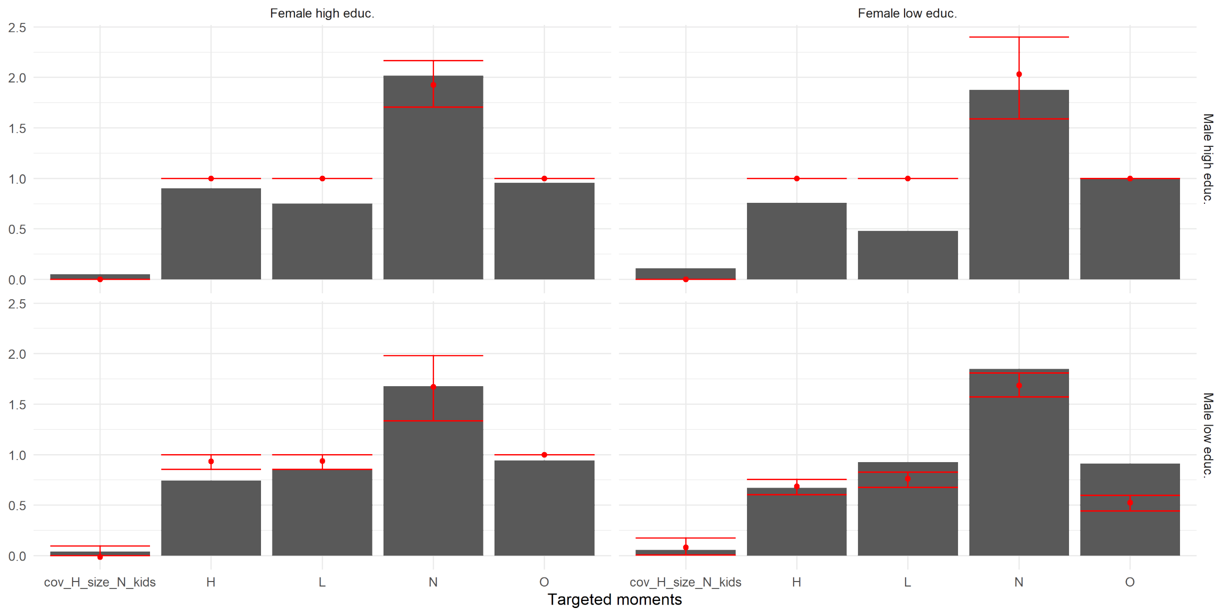
4.2 Model fit

In this subsection, I examine the model fit diagnostics. I use the simulated method of moments (SMM) to estimate the parameters to match a set of moment conditions comprising conditional expected values and covariances. The targets are derived from the Hungarian Household Budget Survey mentioned earlier. I use simple squared distance in the loss function without weights for the moments. I implemented the Cyclic Coordinate Search Algorithm (following Oswald, 2019) to estimate the demand parameters of the model. The algorithm fixes the previous guess of the parameter vector and then converges to a minimum changing only one of the parameters.

I used the following terminal period moments conditional on each combination of parental education of the first cohort: homeownership rate, the fraction of living in large houses, number of children, rural location rate, and covariance between house size and number of children. For the model fit, I used 200 Cohort-0 households without any other cohorts in 150 simulations, as I fixed the house prices at 350,000 HUF and 250,000 HUF for the central and the rural location, respectively, corresponding to the long-run average as we have seen in Figure 6 earlier.

The following Figure 9 displays the in-sample fit of the model; the grey columns show the targeted data moments, while the red error bar shows the mean simulated moment with the 5th and 95th percentile, resulting from 150 simulations. We can see that the model has varying success across the moments. While fertility seems to be well-captured, other aspects often run into corner solutions and overshoot (house size or location for households of highly educated males), or in other instances, undershoot (low education home ownership) the target.

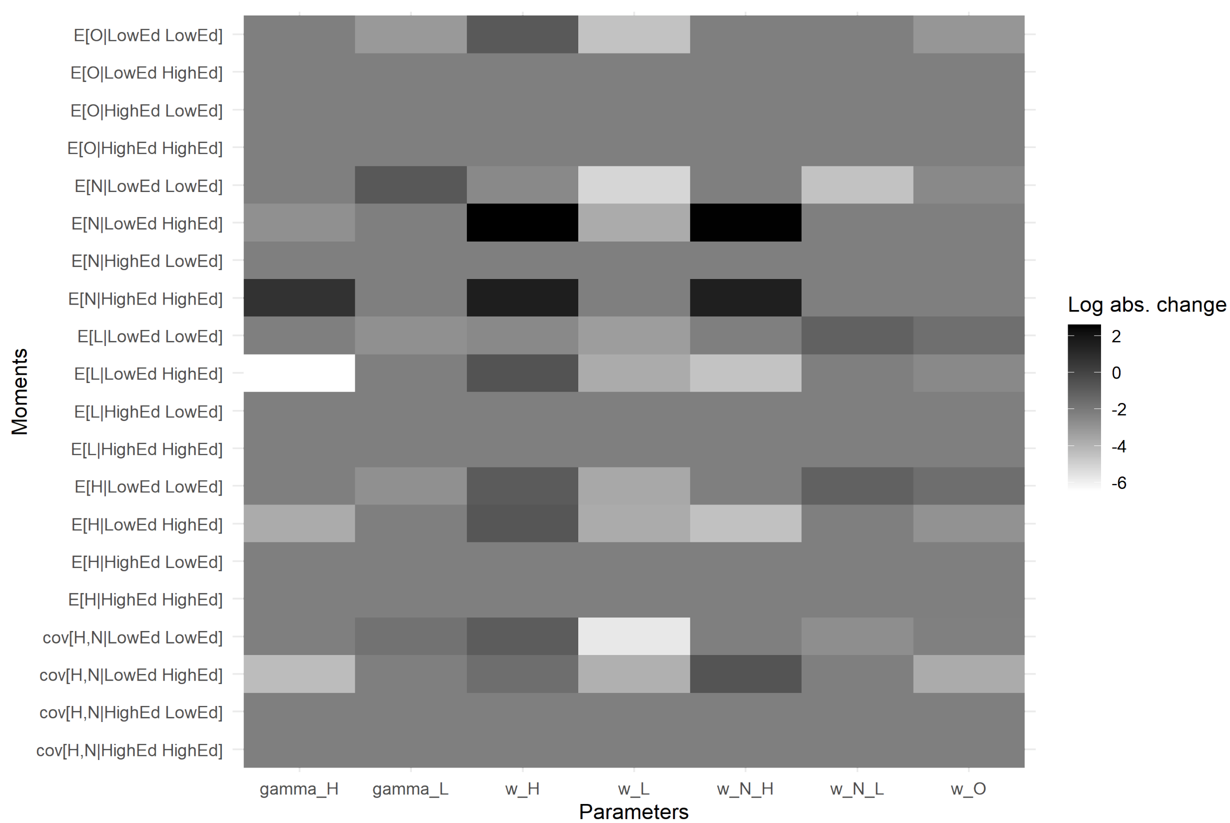
Figure 9: Fit of targeted moments



Note: model fit using 150 simulations of 200 cohort-0 households, grey columns show the target, while the red error bars show the simulated moment with its 5th and 95th percentile.

Identification of the demand parameters can be summarized by the following heatmap of Figure 10, showing the intensity of change in moments responding to a marginal change in parameter values. The results are not surprising. We can see that the CRRA parameters (γ_{ed^F}) and the children parameters ($w^N_{\text{ed}^F}$) react mostly to the expected number of children. House crowdedness parameter w^H to house size and the number of children, while location w^L and ownership w^O affect the moments more weakly but more uniformly.

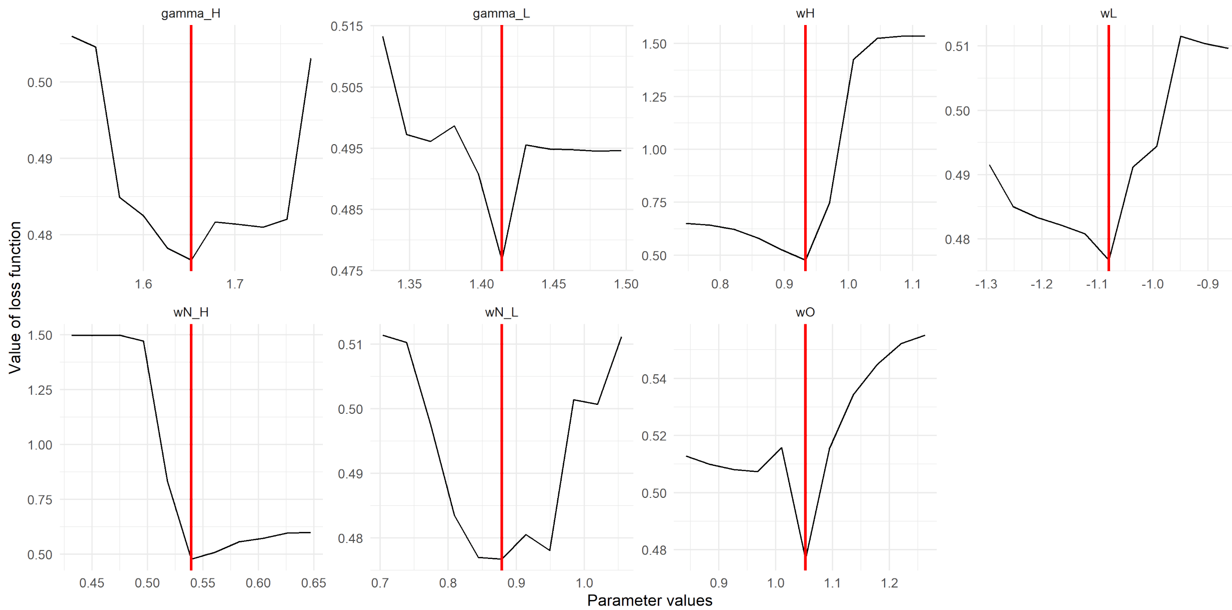
Figure 10: Identification of demand parameters



Note: using 150 simulations of 200 cohort-0 households.

Finally, Figure 11 shows the values of the loss function as a function of the parameters, indicating that, indeed, we have reached a local minimum with the combination of the parameter estimates in an adequately wide neighborhood of the minima.

Figure 11: Loss function by parameters



Note: using 150 simulations of 200 cohort-0 households.

4.3 Parameter estimates

Table 5 presents the parameter estimates and calibrations used for the model simulations. The parameters in the utility function are estimated by the simulated method of moments discussed earlier, while the wage regression parameters are estimated using reduced-form regressions. As discussed earlier, the supply parameters are chosen to minimize the distance from the price levels treated as equilibrium prices under the flexible price regime.

We can see that the γ parameters of the CRRA function are close to the value commonly found in the literature (as discussed by Attanasio et al. (2012), around 1.5). I also find that this parameter is higher for households of females with higher education. The w^N-parameters governing the disutility of being distant from the desired number of children is higher for females with lower education, consistent with the findings of Adda et al. (2017) that early career choices incorporate future fertility plans for females. It might manifest here that those who want more children select into careers with lower wages, hence end up with lower education. The preference parameters regarding housing conditions indicate that given the model is correct, crowdedness (w^H), location (w^L), and homeownership (w^O) all play important roles in the life-cycle behavior of households; however, the estimate for the location parameter is not statistically significant. Households are also found to appreciate homeownership and dislike crowdedness.

The regression parameters of the wage regressions show that wage profiles by age follow the usual quadratic shape on average for both genders. There is a substantial premium for higher education and also for being located in the central area[26].

[26]Note that we abstract away from commuting, which could potentially be very important, here, the estimates refer to the location of the firm sites.

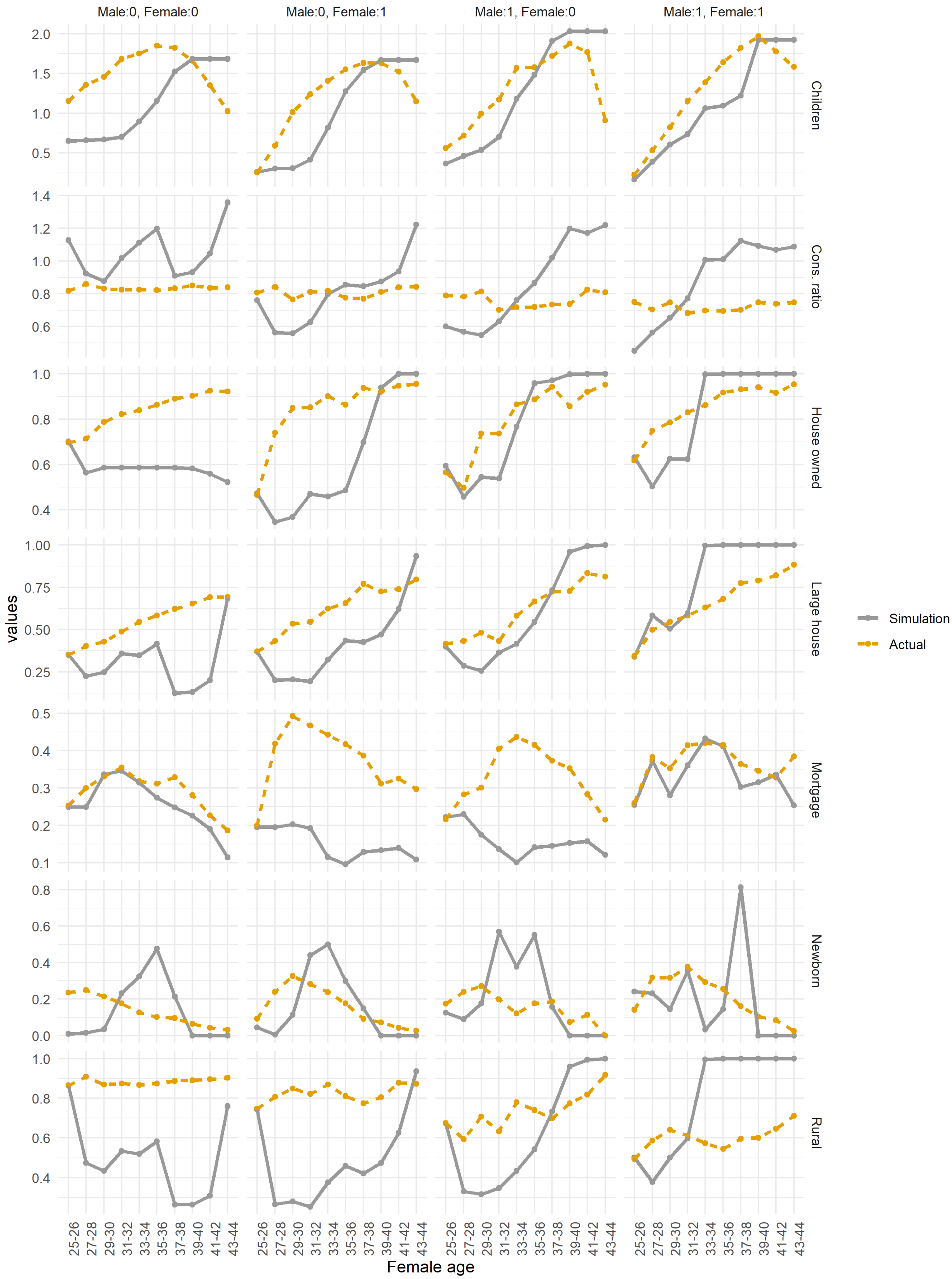
Table 5: Parameter estimates and calibrations

Parameter	Value, Estimate (S.E.)	Source
Utility function		
β	0.91	Set manually (biannual case)
γ_l	1.4140 (0.0599)	Estimated (SMM)
γ_h	1.6524 (0.0299)	Estimated (SMM)
w_l^N	0.8790 (0.1021)	Estimated (SMM)
w_h^N	0.5394 (0.0225)	Estimated (SMM)
w^H	0.9330 (0.0212)	Estimated (SMM)
w^L	-1.0790 (0.8978)	Estimated (SMM)
w^O	1.0522 (0.1242)	Estimated (SMM)
Budget constraint		
ir	0.10	HNB (2019) (biannual case)
δ	0.50	Set manually
ρ	0.05	Set manually
$\beta_0^{W,M}$	11.78 (0.0062)	Estimated (reduced form)
$\beta_1^{W,M}$	0.0219 (0.0005)	Estimated (reduced form)
$\beta_2^{W,M}$	-0.0005 (1.54e-05)	Estimated (reduced form)
$\beta_3^{W,M}$	0.913 (0.0071)	Estimated (reduced form)
$\beta_4^{W,M}$	-0.101 (0.0057)	Estimated (reduced form)
$\beta_0^{W,F}$	11.77 (0.0066)	Estimated (reduced form)
$\beta_1^{W,F}$	0.0170 (0.0006)	Estimated (reduced form)
$\beta_2^{W,F}$	-0.0004 (1.60e-05)	Estimated (reduced form)
$\beta_3^{W,F}$	0.772 (0.0071)	Estimated (reduced form)
$\beta_4^{W,F}$	-0.183 (0.0062)	Estimated (reduced form)
p_c	1,552.5 (in 1,000 HUF)	HCSO (2016)[27]
$W_{\min}$	960 (in 1,000 HUF)	Based on minimum wage in Hungary
Supply parameters		
α^0	0.0274	Calibrated to 2004-2014 house prices
α^1	0.0397	Calibrated to 2004-2014 house prices

4.4 Model validation

As we fit the model on the moments of the terminal values, we can use the previous periods to validate how well the model captures the dynamics of the key variables over the life cycle. Figure 12 displays how our main variables compare with their actual counterparts. We can see some weaknesses, most importantly concerning rural vs. central location choice, which could explain why the parameter estimate governing that aspect is not statistically significant. While in actuality, highly educated households choose to reside in the central location more than lower education households, in this model, we get the opposite. The central location provides higher earnings, so lower-education households are forced live in the central location comparatively more often. At the same time, the central location also gives disutility to households according to the parameter estimates. It results in households with higher education choosing to reside in the rural area without exception in the model.

Figure 12: Evolution of key variables over the life cycle, simulation vs. actual



Note: using 150 simulations of 200 cohort-0 households and values based on the HKÉF data of 2004-2014.

5 Results of the policy simulations

Using the parameter estimates, I simulated the model for six different scenarios 150 times, with 150 Cohort-0, Cohort-1, and older households over ten biannual periods. House prices are endogenously evolving in all of them, possibly counteracting the intended policy effects. Then I study how fertility and housing variables evolve under these scenarios and show comparisons of household welfare in a partial equilibrium setting. The scenarios were the following:

1. No allowance (baseline)
2. Allowance available (Family Housing Allowance for larger houses)
3. Extra supply in housing without allowance (+200% in the α parameters)
4. Lower interest rate without allowance (3.5% instead of 5%)
5. Lower down payment requirement without allowance (40% instead of 50% required)
6. Lower interest rate with allowance (3.5% instead of 5% rate, with Family Housing Allowance)

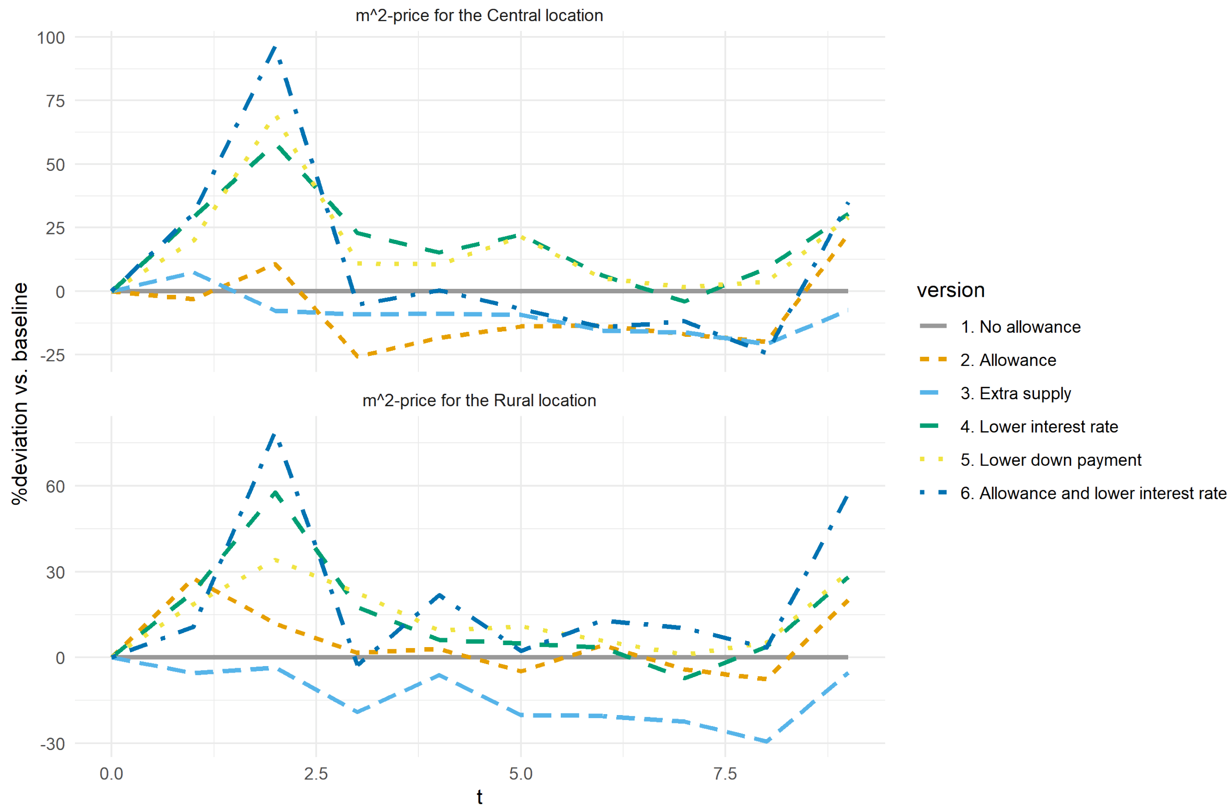
I included a combined monetary and fiscal policy scenario, 'Lower interest rate with allowance', to reflect that the Hungarian National Bank decreased interest rates substantially starting in 2013. So instead of the 5% interest rate assumed during the model estimation, which is more relevant for the pre-2014 period, I use 3.5% to represent the post-2014 period more accurately. Scenarios 'Allowance' and 'Lower interest rate' with and without the government allowance enable us to disentangle the potential effects of the policy on housing and fertility. The scenarios 'Extra supply' and 'Lower down payment' represent alternative housing market policies that directly target the housing market without fertility incentives, allowing us to examine the effects of simply alleviating housing-related constraints on the households.

5.1 Effect on the housing market

In this exercise, I assess the evolution of house prices compared to the baseline scenario. Note that even in the baseline scenario, I cannot reproduce constant prices at the target values described earlier with only two parameters (α); only the average over the 10-period (20-year) frame will be close to those values. Otherwise, there are fluctuations driven by the young households' life-cycle (in Appendix, Figure A8). However, life-cycle fluctuations are similar for all scenarios, so that the differences can be interpreted as the effect of the policies.

First, let us consider the evolution of house prices under the proposed scenarios in Figure 13, showing %-deviation compared to the baseline. We can immediately observe that lower interest rates and down payment ratios will result in higher house prices in both the central and the rural locations (50-80% for the central and 30-60% for the rural) for the first four years (two periods). More importantly, we can see that besides an around 30% short-term increase in rural area house prices, the government allowance alone does not induce such significant changes close to what we would observe in the actual data. However, the allowance combined with a lower interest rate does produce around 100% increase in house prices for the central and 75% for the rural areas, which are somewhat close to the actual data points. However, after three to four periods, house prices will not differ much from the baseline scenario values. So the deviation is only temporary in this model (possibly due to only being able to simulate two cohorts, implicitly enforcing this adjustment mechanism). On the contrary, with the extra supply scenario, we can naturally drive down the prices even in the long run. Still, even an around 200% permanent increase in new housing decreases house prices by only 20-30%.

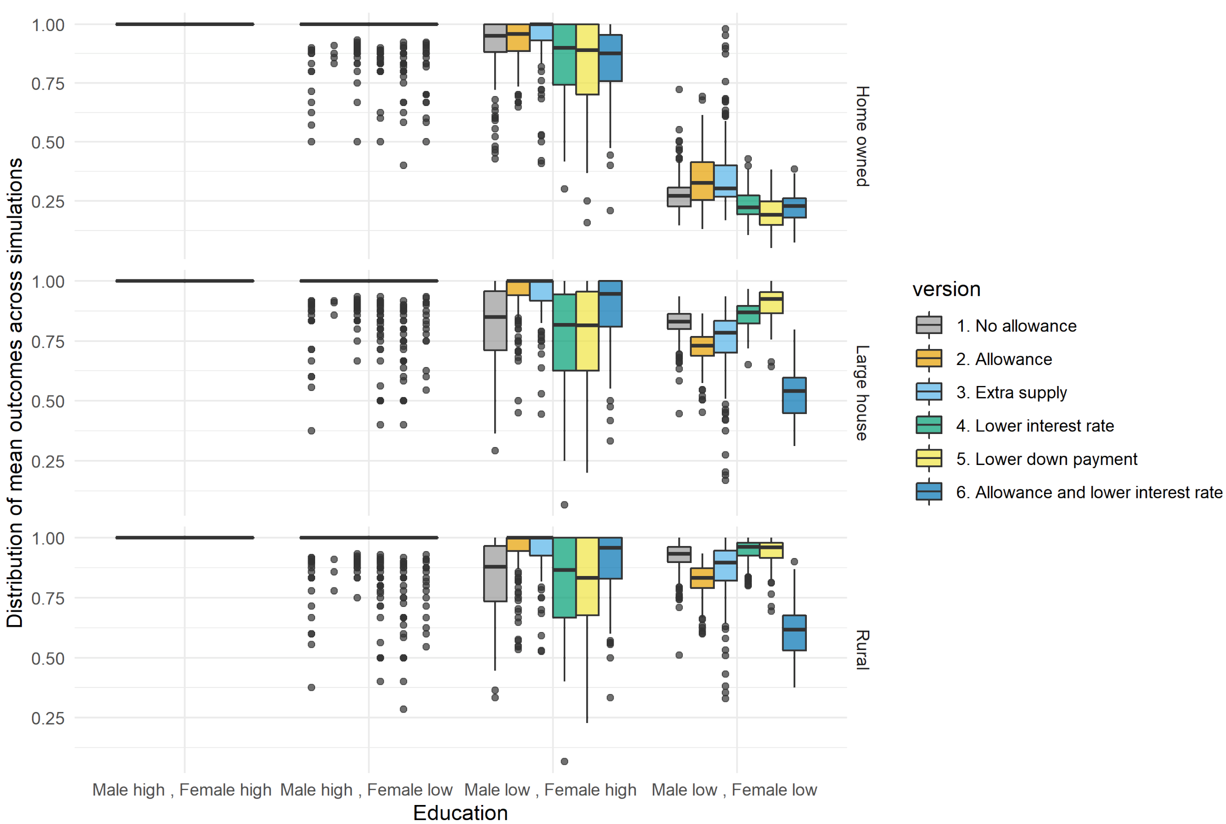
Figure 13: Evolution of house prices compared to the baseline scenario



Note: using 150 simulations of 150 Cohort-0, Cohort-1 and 'old' households. One period on the horizontal axis represents two years.

The housing-related terminal outcomes of households are displayed in Figure 14 (the evolution of the variables can be found in the Appendix, Figure A5). It is easy to see that there is no effect for households with high male education, as we run into a corner solution for most of these households. However, it suggests that housing outcomes in this model are related to household income primarily driven by male income. The results are mixed for households with low male education; I will focus on the most numerous low-male and low-female education groups. Regarding ownership, we can see that the homeownership ratio does not change much due to the scenarios. While with allowance and extra supply, it is slightly higher, in the other scenarios, it is somewhat lower. The fraction living in a large house does change substantially due to the policies. The Housing Allowance counter-intuitively seems to decrease the fraction of these poorer households living in a larger house, especially if interest rates are lower, which induces the greatest elevation in house prices. Living in a large house is higher than the baseline if down payment requirements are lower, while they are around the same for the other scenarios. The fraction living in the rural location also decreases due to the allowance, even more under lower interest rates. This finding reflects the elevated house prices and that households can afford only smaller homes when moving into the central location to receive higher salaries.

Figure 14: Effect of scenarios on end-of-life-cycle housing outcomes



Note: using 150 simulations of 150 Cohort-0, Cohort-1, and 'old' households, based on the results of Cohort-0, the results for Cohort-0.

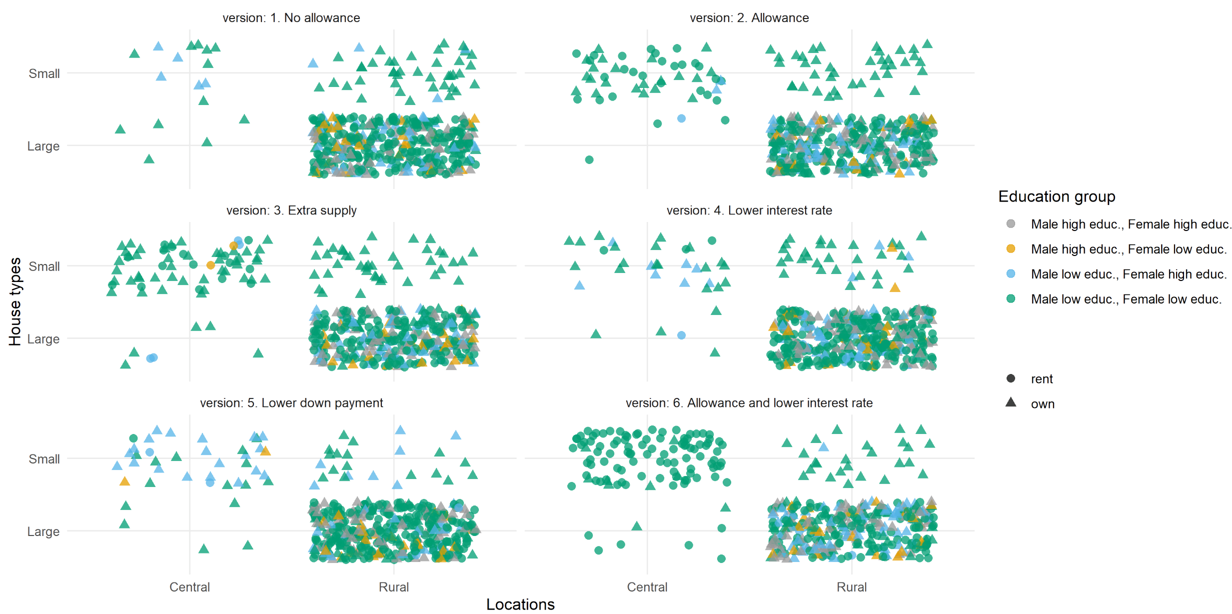
We can examine the sorting into house sizes and locations of households under our scenarios by education groups. Table 6 shows how households are distributed on average within their education group across sizes and locations, while Figure 15 illustrates the distribution of households with information on ownership status as well. We can again see that high-male education households live dominantly in large rural houses under any scenario, while the distribution of low-male education households varies by the circumstances. Most importantly, under the 'Allowance with lower interest rate' scenario, many more of the low male and low female education households live in small central area houses than under the other scenarios, which spot is the least desired according to the estimated preference parameters. At the same time, our figure shows that these households rent and do not own apartments in the center. This suggests the Allowance policy combined with low interest rates harms the poorest households' housing conditions, while neither would have such an impact.

Table 6: Household sorting into house size and location, by education

Size	Location	Education	Baseline	Allowance	Extra supply	Lower interest rate	Lower down payment	Allowance and lower interest rate
Large	Rural	Male high educ., Female high educ.	1.00	1.00	1.00	1.00	1.00	1.00
Large	Central	Male high educ., Female low educ.	0.00	0.00	0.00	0.01	0.01	0.01
Large	Rural		0.97	1.00	0.98	0.94	0.94	0.97
Small	Central		0.03	0.00	0.01	0.04	0.04	0.02
Small	Rural		0.00	0.00	0.01	0.01	0.01	0.00
Large	Central	Male low educ., Female high educ.	0.01	0.02	0.01	0.01	0.02	0.03
Large	Rural		0.81	0.92	0.93	0.76	0.75	0.86
Small	Central		0.16	0.04	0.04	0.17	0.17	0.08
Small	Rural		0.03	0.02	0.01	0.06	0.06	0.04
Large	Central	Male low educ., Female low educ.	0.01	0.01	0.01	0.01	0.01	0.01
Large	Rural		0.80	0.71	0.73	0.85	0.89	0.52
Small	Central		0.07	0.17	0.13	0.05	0.05	0.38
Small	Rural		0.12	0.11	0.13	0.10	0.05	0.09

Note: the author's calculation based on 150 simulations of the model Cohort-0.

Figure 15: Household sorting into house size, location by ownership status and education

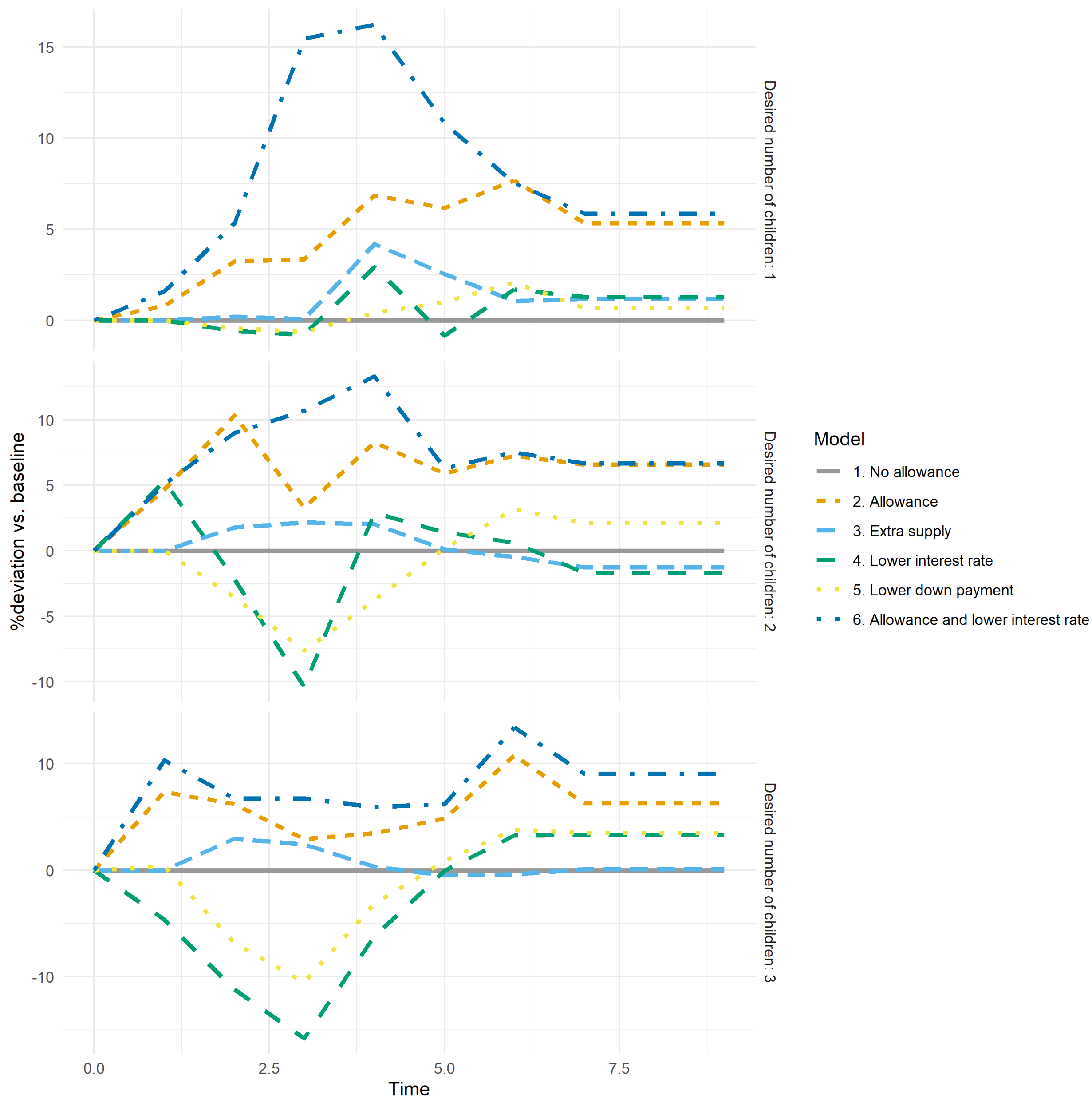


Note: the author's calculations based on three simulations of 150 Cohort-0, Cohort-1, and 'old' households, to allow for visibility of the results.

5.2 Effect on fertility and welfare

One point of focus in this analysis is to evaluate the Family Housing Allowance policy's potential long-run effects on the completed fertility outcomes of households vs. timing effects. Figure 16 shows the %-deviation compared to the baseline scenario, in the number of children for Cohort-0's life-cycle, by the number of desired children. In this model setting, the Allowance results in an approximately 5-10% increase in completed fertility by the end of the life cycle. The policy also alters timing, as we can see that children are born earlier in the life cycle under the Allowance policy (hence the larger difference at the earlier periods). Other policies also affect fertility but to a lower degree. Lower down payment seems to cause an around 2-5% increase in completed fertility. A lower interest rate increases fertility for households that want one or three children but decreases fertility for those that want two children. The extra supply scenario causes a slight rise in fertility for households that want one child but a small decrease for those that want two, with no change for those that would like three.

Figure 16: Evolution of the total number of children compared to the baseline scenario, by the desired number of children



Note: using 150 simulations of 150 Cohort-0, Cohort-1 and 'old' households, the results of Cohort-0. One period on the horizontal axis equals two years.

We can also examine which education groups change their fertility due to different policies

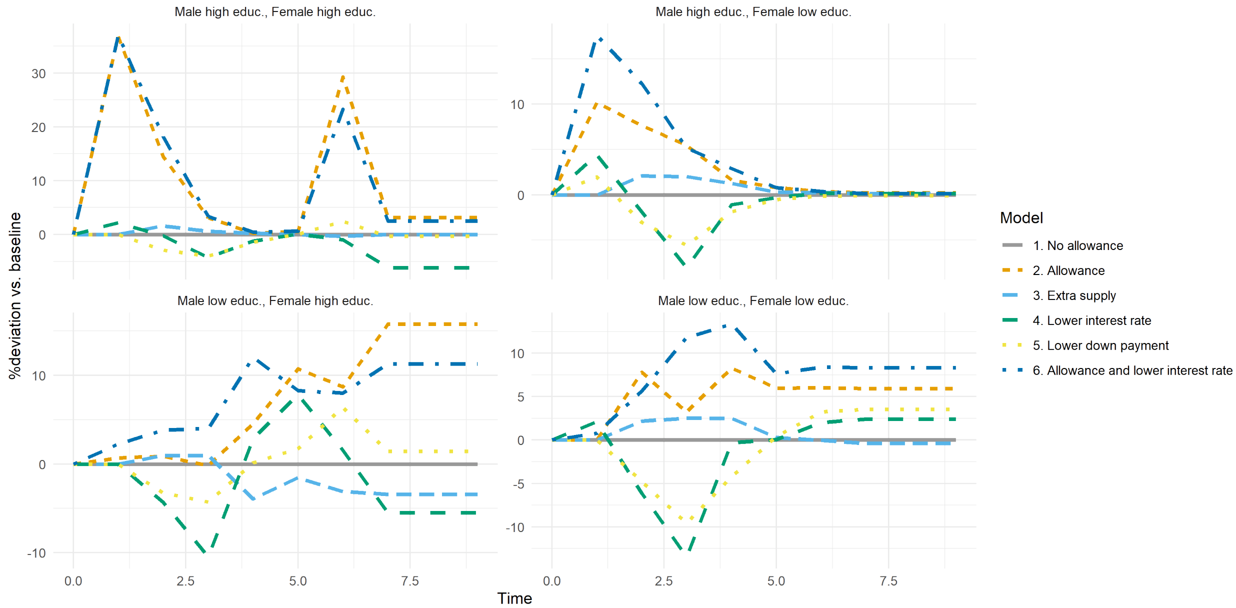
— Table 7 and Figure 17 display these results. Households with high education increase their completed fertility by 0.05 (around 2.5%) child per family as a response to either allowance scenario. In contrast, their fertility decreases substantially (-0.12 child per family) under the lower interest rate scenario. Households with lower education react somewhat differently: they increase their completed fertility more as a reaction to the allowance policies (0.11 and 0.15 child per family, or around 6-8% respectively). However, their fertility is unchanged or slightly changed under all other policies. Changes in the timing of births depend on household education as well. Households with high male education bring their births forward in time due to the Allowance, but as we saw, it does not imply much in completed fertility. In comparison, lower education households increase their completed fertility more with the allowance, while their reaction in timing is lower.

Table 7: Average completed fertility by education

Education group	Baseline	Allowance	Extra supply	Lower interest rate	Lower down payment	Allowance and lower interest rate
Male high educ., Female high educ.	1.9282	1.9886	1.9279	1.8090	1.9215	1.9768
	(0.0126)	(0.0123)	(0.0126)	(0.0137)	(0.0128)	(0.0126)
Male high educ., Female low educ.	2.0596	2.0636	2.0620	2.0620	2.0580	2.0620
	(0.0172)	(0.0173)	(0.0173)	(0.0172)	(0.0173)	(0.0174)
Male low educ., Female high educ.	1.5697	1.8176	1.5160	1.4834	1.5920	1.7471
	(0.0154)	(0.0153)	(0.0155)	(0.0155)	(0.0154)	(0.0156)
Male low educ., Female low educ.	1.8219	1.9301	1.8146	1.8663	1.8860	1.9739
	(0.0061)	(0.0055)	(0.0062)	(0.0061)	(0.0060)	(0.0054)

Note: using 150 simulations of 150 Cohort-0, Cohort-1 and 'old' households, the results of Cohort-0. One period on the horizontal axis equals two years.

Figure 17: Evolution of the total number of children compared to the baseline scenario, by education



Note: using 150 simulations of 150 Cohort-0, Cohort-1 and 'old' households, the results of Cohort-0. One period on the horizontal axis equals two years.

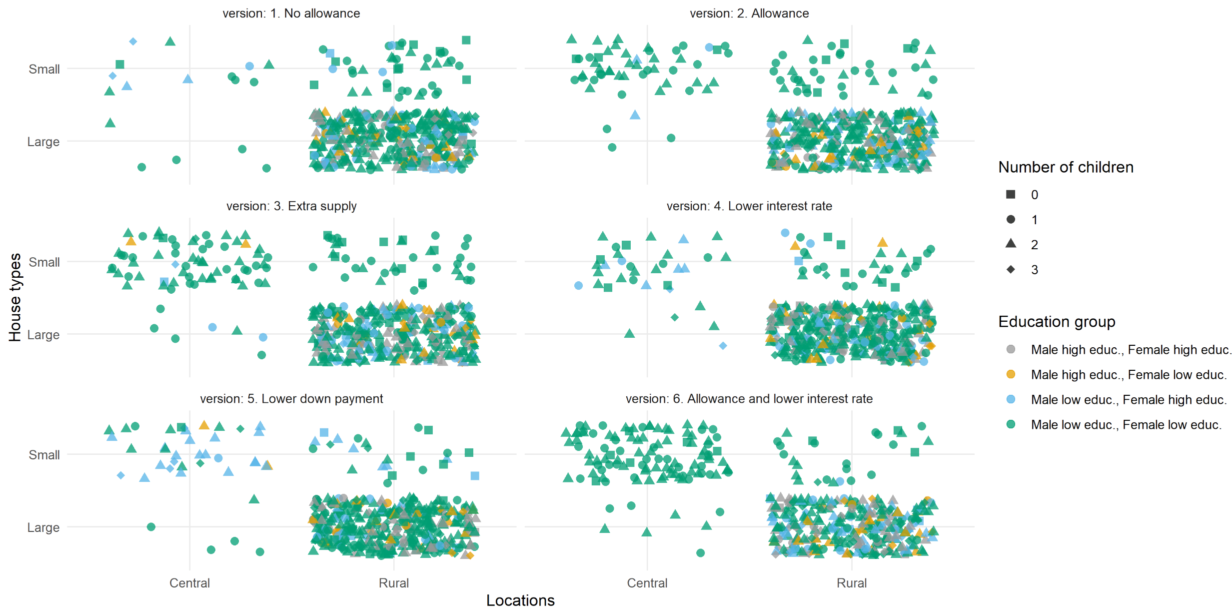
Finally, we can look at how families sort into housing conditions under different scenarios based on their fertility outcomes: Table 8 shows the distribution within the final number of children across house sizes and locations. In contrast, Figure 18 illustrates this with information on the households' education. (The evolution of the variables can be found in the Appendix, Figure A6) We can see that policies do not affect where families without or with three children live. While families without children settle in large or small rural houses, families with three live in large rural houses almost exclusively. The policies, however, change where families with one or two children end up living. Compared to the baseline, in the scenarios with the Family Housing Allowance, more of them reside in small central location houses (30% point increase), and most of those would otherwise have lived in large rural dwellings in the baseline scenario. It suggests that according to this model, those families who do not end up with three children are somewhat worse off regarding their housing conditions. The figure also indicates that these families that end up in the central location are households with low education.

Table 8: Household sorting into house sizes and location, by number of children

Size	Location	Children	Baseline	Allowance	Extra supply	Lower interest rate	Lower down payment	Allowance and lower interest rate
Large	Central	0	0.02	0.01	0.02	0.01	0.02	0.01
Large	Rural		0.70	0.65	0.73	0.73	0.70	0.69
Small	Central		0.05	0.05	0.05	0.04	0.04	0.04
Small	Rural		0.23	0.29	0.20	0.22	0.25	0.26
Large	Central	1	0.02	0.02	0.02	0.01	0.03	0.03
Large	Rural		0.72	0.47	0.66	0.80	0.83	0.40
Small	Central		0.09	0.29	0.14	0.05	0.06	0.38
Small	Rural		0.17	0.22	0.18	0.14	0.08	0.18
Large	Central	2	0.01	0.00	0.00	0.01	0.01	0.01
Large	Rural		0.89	0.85	0.86	0.89	0.92	0.64
Small	Central		0.06	0.10	0.08	0.06	0.05	0.32
Small	Rural		0.05	0.04	0.05	0.05	0.02	0.04
Large	Central	3	0.00	0.00	0.00	0.01	0.00	0.00
Large	Rural		0.91	0.99	0.90	0.90	0.90	0.97
Small	Central		0.08	0.00	0.08	0.08	0.08	0.02
Small	Rural		0.01	0.00	0.01	0.02	0.02	0.01

Note: the author's calculation based on 150 simulations of the model to allow for visibility of the results.

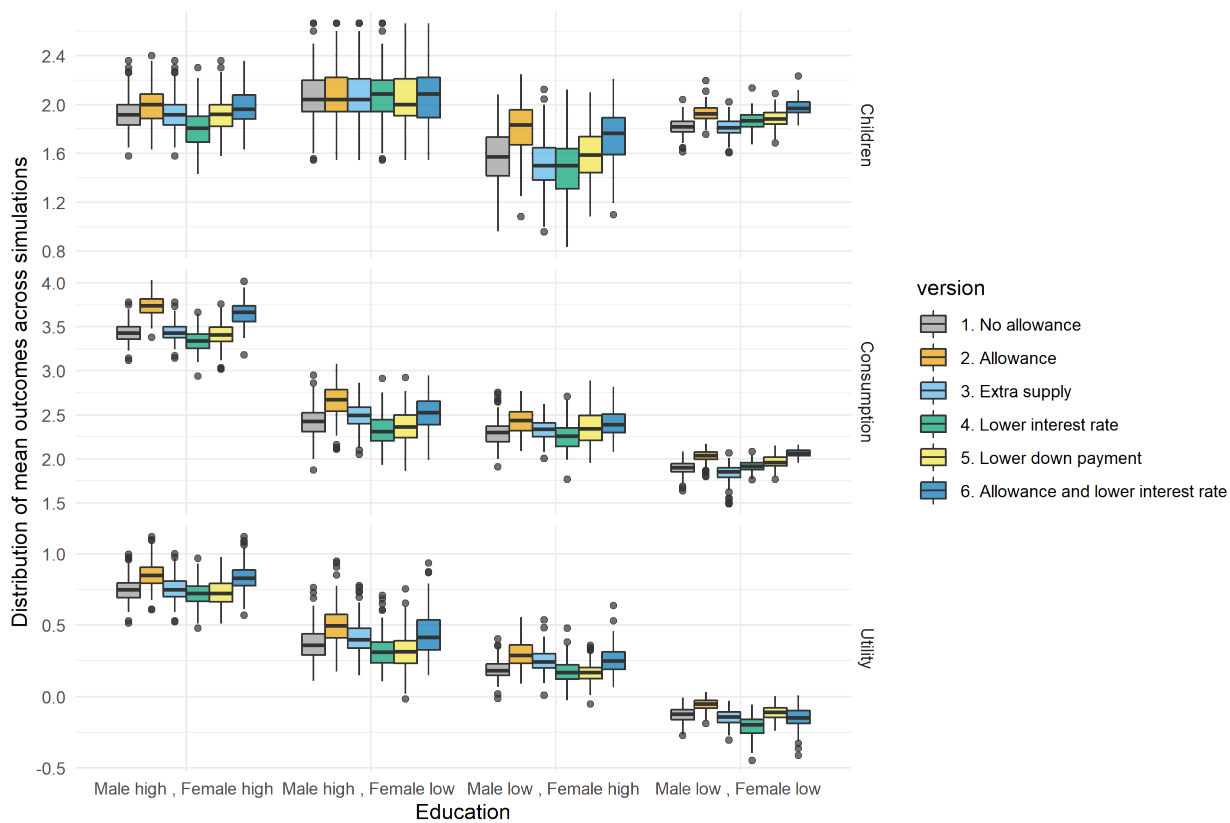
Figure 18: Household sorting into house size, location by the number of children and education, under three scenarios



Note: the author's calculations based on three simulations of 150 Cohort-0, Cohort-1, and 'old' households, to allow for visibility of the results.

Finally, we can also compare the scenarios directly by calculating the average discounted utilities realized over the life cycles of the households. Figure 19 shows these utilities, along with showing average lifetime consumption and then again the completed fertility, now as the distribution of averages over simulations. We can also see that for households with high education for at least one parent, the Allowance improved average utilities. In contrast, all other policies leave it more or less unaffected. However, lower interest rates seem to decrease utility substantially for households where both males and females have lower education. So much so that even with the Allowance, the net effect is not positive (while Allowance without lower interest rates seems to have a positive utility effect). I have already discussed that housing conditions of low-education households worsen under lower interest rates, which is then compensated by increased consumption and the number of children to have a net neutral effect. The figure shows that for all groups, non-surprisingly, the available government subsidy will result in higher consumption, while the other policies leave it unaffected.

Figure 19: Effect of scenarios on non-housing outcomes



Note: the author's calculations based on 150 simulations of 150 Cohort-0, Cohort-1 and 'old' households, the results for Cohort-0.

6 Discussion and conclusion

In this paper, I constructed a life-cycle model of household behavior focusing on fertility, location, and housing choices, with unobserved heterogeneity in the preferences for the desired number of children and endogenously evolving house prices. Using the period of 2004-2014, I estimated the parameters characterizing the preferences of the households and then used them to analyze the effects of different policy scenarios on the housing conditions and completed fertility of households. I concentrated on the potential impact of the Hungarian Government's Family Housing Allowance program, which has been running since 2015.

Without additional changes, the Allowance program itself cannot explain the house price increases observed in Hungary after its introduction. However, combining it with lower interest rates (as these policies indeed coexisted in the relevant time frame) induces a house price evolution relative to the baseline scenario comparable to the one observed after 2015 in Hungary. According to this model, the combination of the fiscal and monetary policy jointly could be responsible for the house price increases, possibly negating some of the welfare effects of the policy.

The model suggests that poorer households (represented by both parents having attained a non-tertiary education level) were affected ambivalently by the policy, mainly driven by the counteracting negative impact of higher house prices. While consumption and the number of children seem to respond positively to the policy incentives, their housing conditions worsen in the long run. It occurs because many have to move to the less desirable central location and rent small apartments instead of living in self-owned, larger rural houses. Comparatively, households with higher education seem to benefit from the policy, seemingly without significant drawbacks. These findings do not seem to contradict the recent brief assessment of HNB (2021), which suggests that the policy helped families with three children but did not raise housing availability for other family types.

As future improvements and extensions of this model, I will attempt to resolve the problems of model fit regarding rural and central location across households with different educational backgrounds. Furthermore, a drawback of the approach I chose is that when allowed to fluctuate endogenously, house prices, even in the baseline scenario, are far from stable, which would further strengthen the conclusions of this study.

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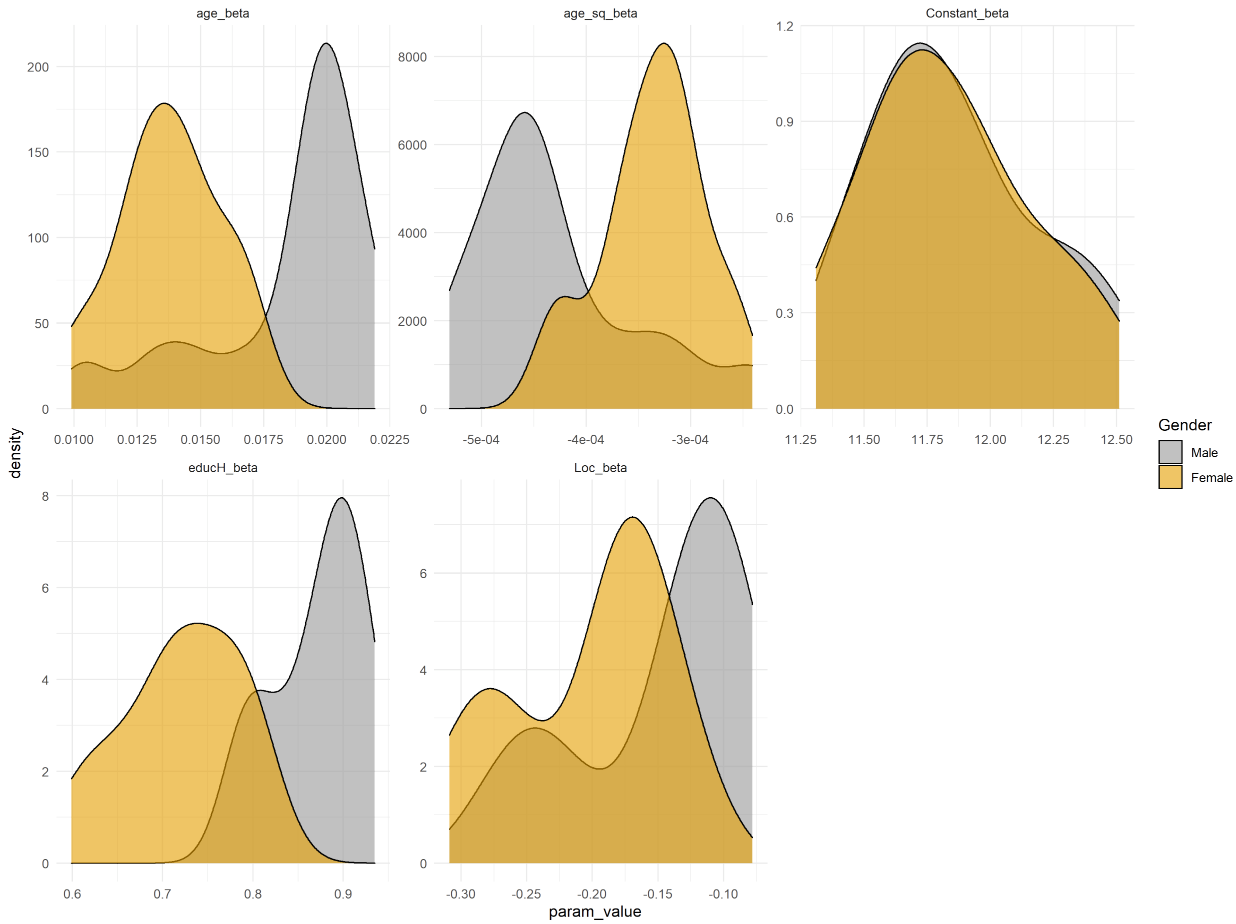
A Appendix

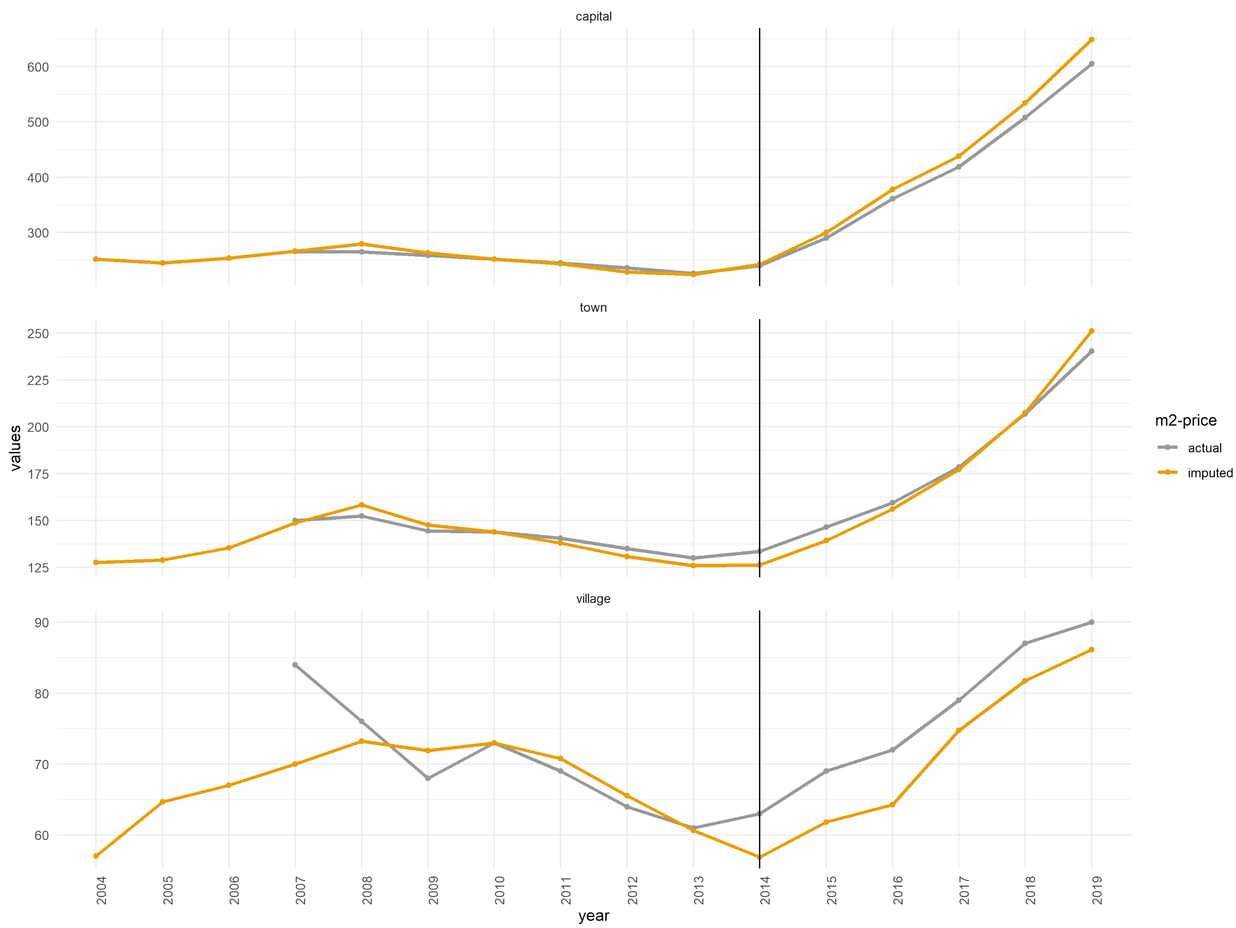
All additional figures and programs that belong to this paper are attached to this document.

Figure A1: Estimated densities of the estimated wage regression parameters, 2004-2018



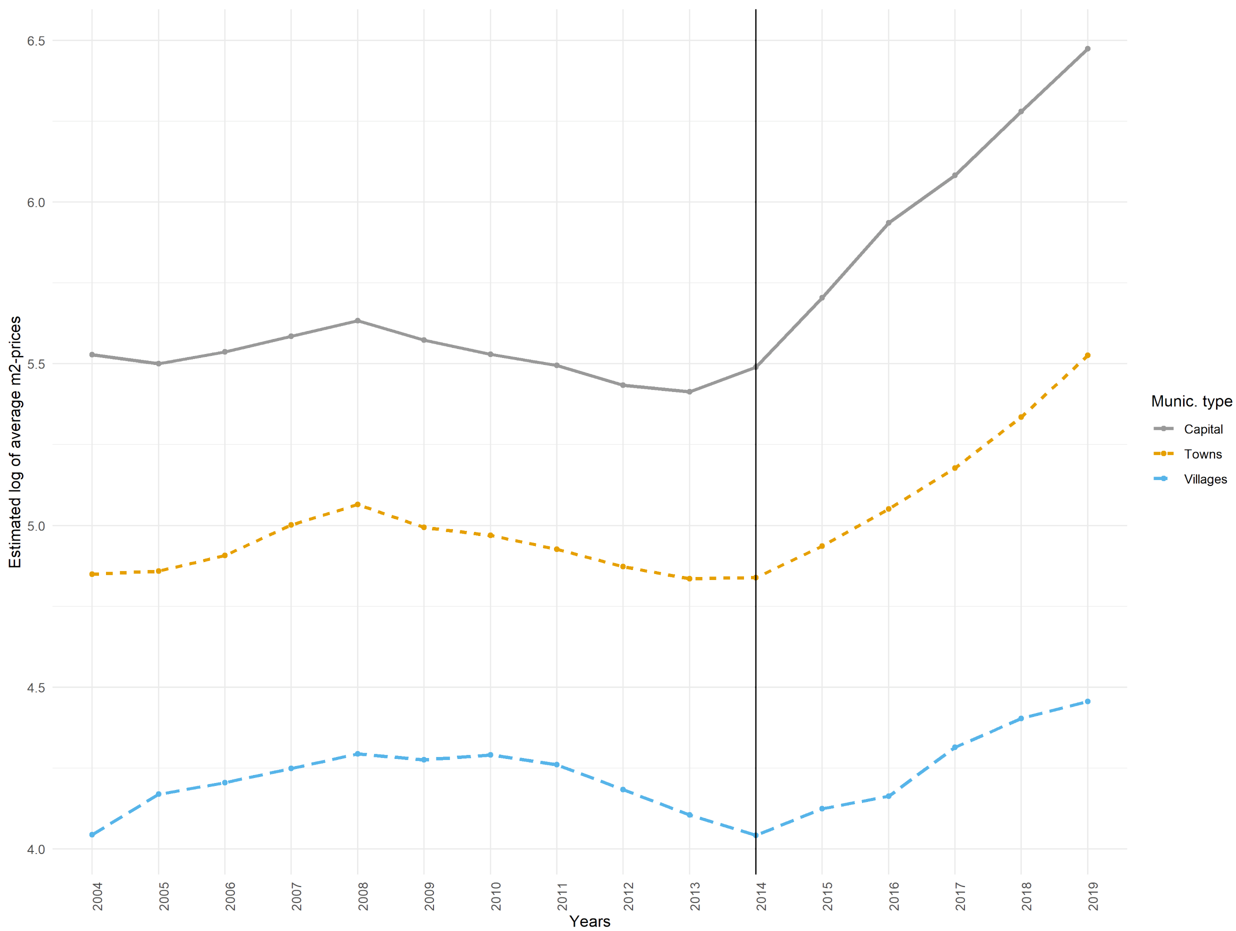
Note: the author's calculations based on the regression estimates from the Wage Survey data of Hungary, 2004-2018.

Figure A2: Imputed m^2-prices compared to available estimates



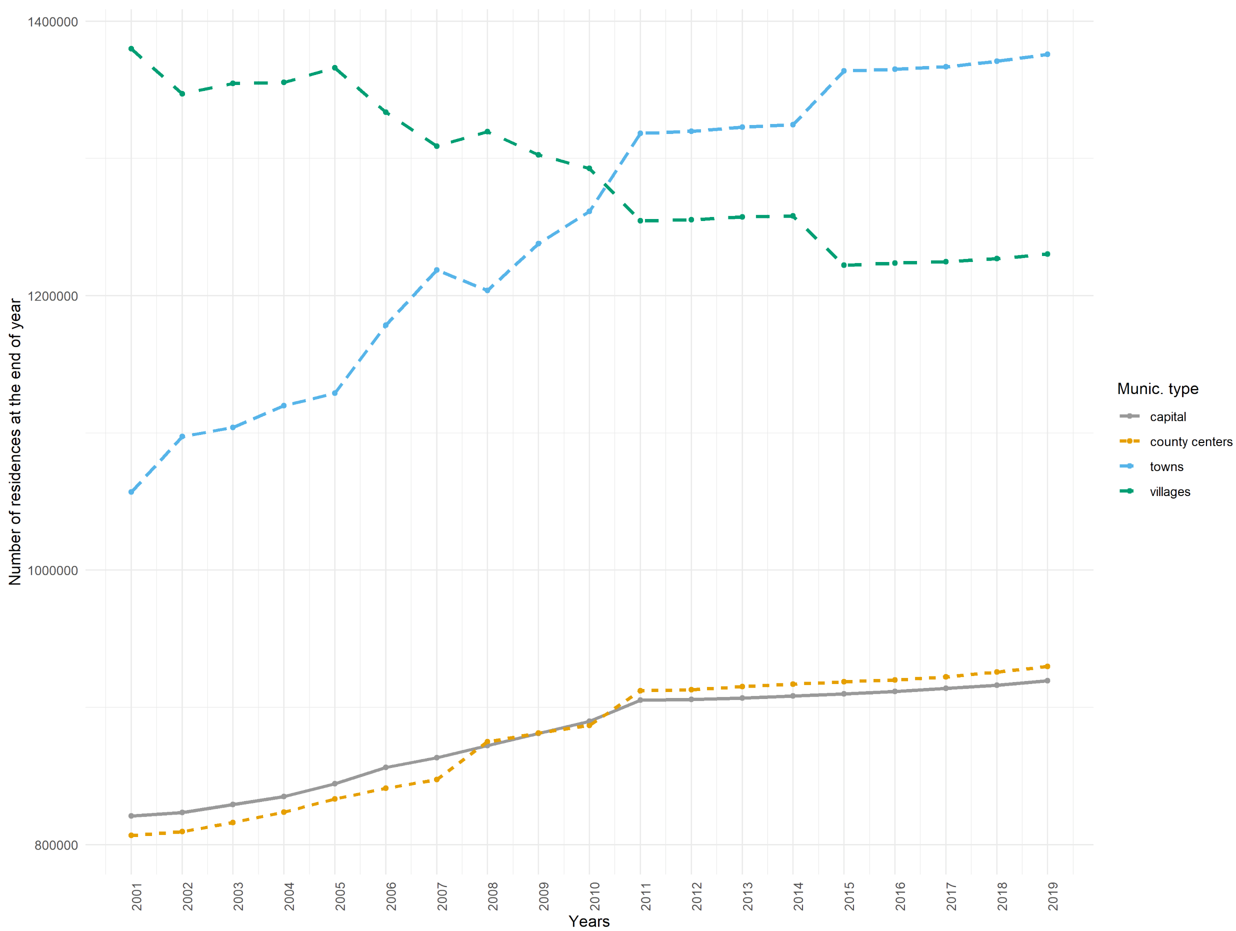
Note: the author's calculations based on the publicly available data of the Hungarian National Bank (HNB) and the Hungarian Central Statistical Office (HCSO). The earliest average m^2-price estimates of the HCSO are only available from 2007, while the HNB publishes nominal and real house price indices for earlier years. I use the two sources of information to estimate the pre-2007 prices, using 2010 as the baseline.

Figure A3: Logarithm of yearly average m^2-prices in 1000s of HUF, by type of municipality, 2004-2019



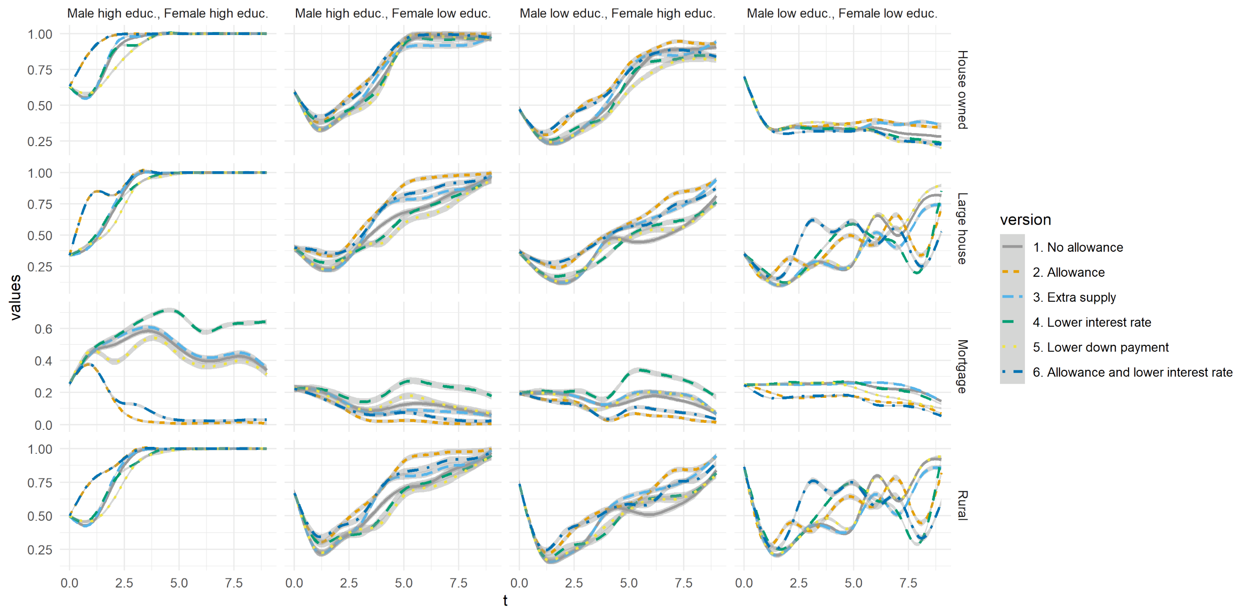
Note: the author's calculations based on the nominal house price index of the Hungarian National Bank and house price estimates of the Hungarian Central Statistical Office.

Figure A4: Number of houses at the end of the year, by type of municipality, 2001-2019



Note: based on the publicly available data of the Hungarian Central Statistical Office.

Figure A5: Evolution of housing variables under different scenarios

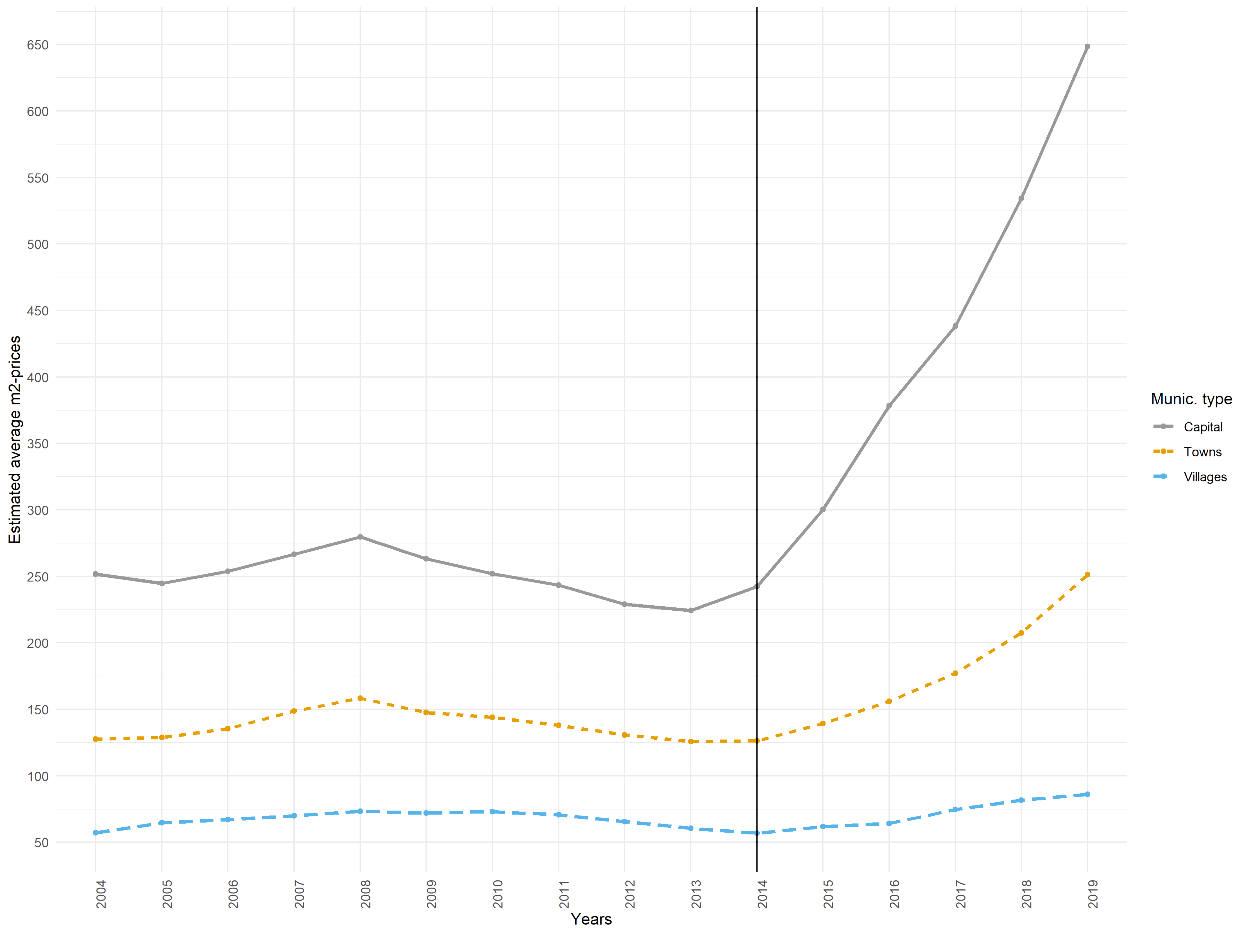


Note: the author's calculations based on 150 simulations of the model. One period on the horizontal axis equals two years.

Figure A6: Evolution of non-housing variables under different scenarios

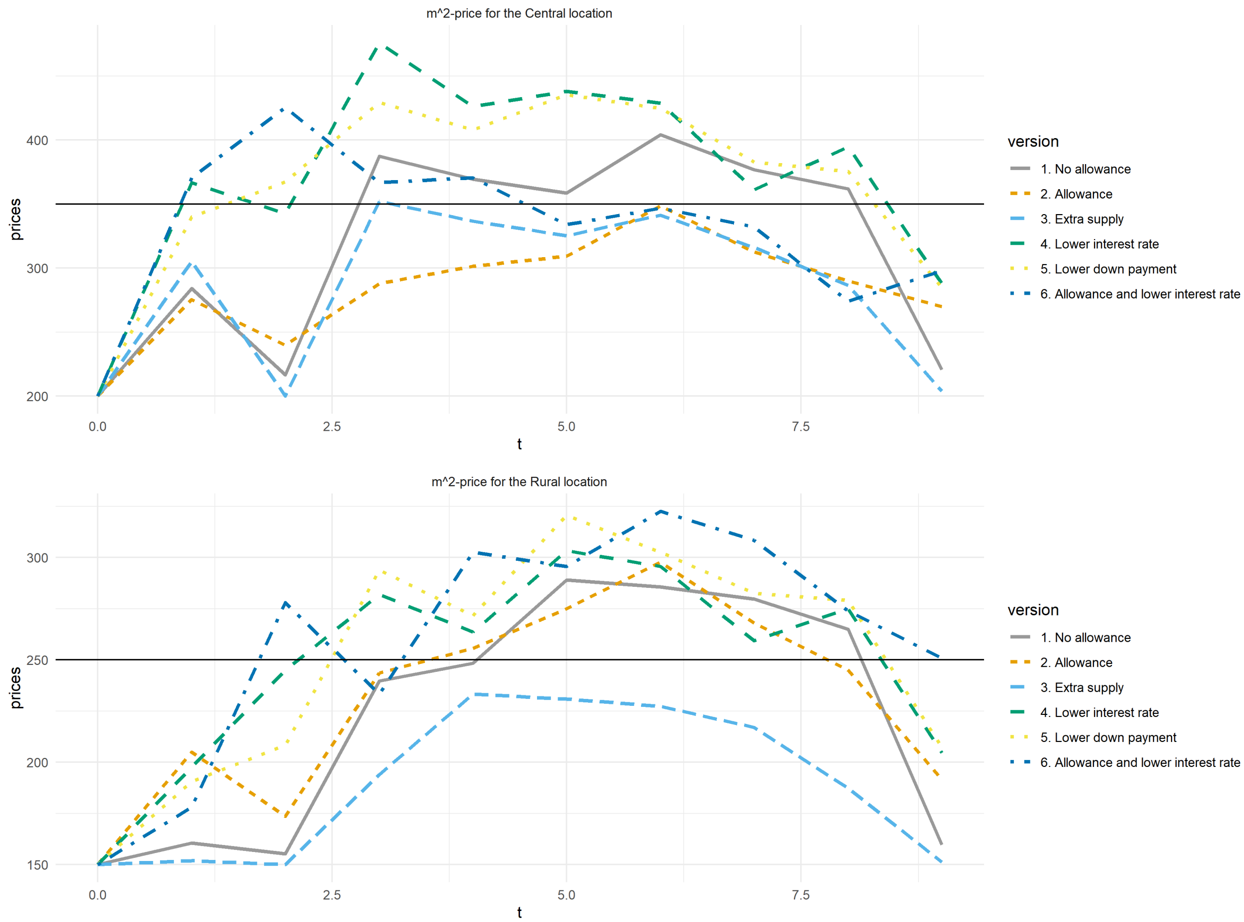
Note: the author's calculations based on 150 simulations of the model. One period on the horizontal axis equals two years.

Figure A7: Average m^2-prices in 1000s of HUF, by type of municipality, 2004-2019



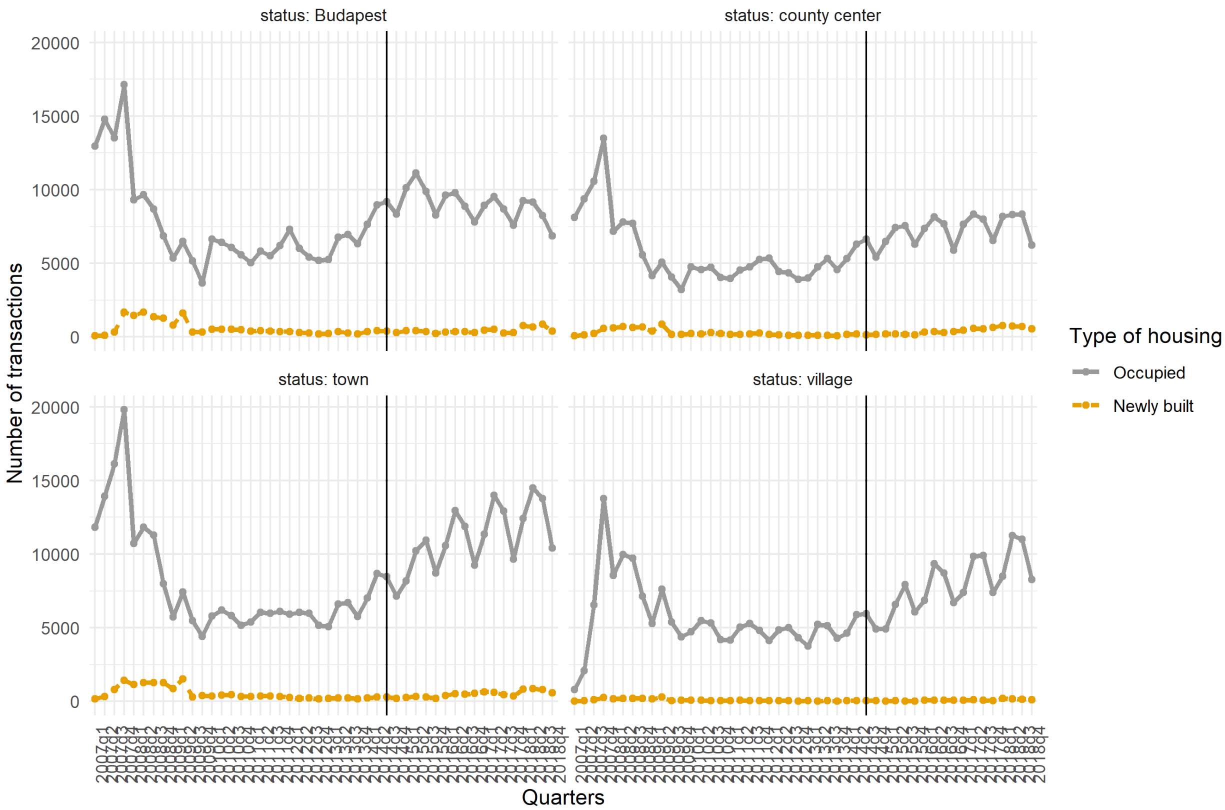
Note: the author's calculations based on the nominal house price index of the Hungarian National Bank and house price estimates of the Hungarian Central Statistical Office.

Figure A8: Average m^2-prices for the scenarios



Note: the author's calculations based on 150 simulations of the model. One period on the horizontal axis equals two years. Targets of the prices are indicated with the horizontal lines at 350 for the central and 250 for the rural area.

Figure A9: Number of housing transactions by type of municipality and house, 2007q1-2018q4



Note: based on the publicly available data of the Hungarian Central Statistical Office.